Addressing Private Practice in Public Hospitals

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Addressing Private Practice in Public Hospitals

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Abstract: This paper proposes a theoretical analysis of the private provision of care within public hospitals and assesses its impact on the quality and cost of healthcare. We also capture this policy's impact on the number of outpatients that are seen and the number that are cured. We show that the private income gathered by consultants engaged in dual practice has a negative impact on the level of care being provided as it incentivises consultants to focus on the number of patients seen. However, the private fees generate lower healthcare costs. Hence the removal of private practice in public hospitals is only optimal when the benefit associated with curing patients is large enough. The impact on waiting lists is ambiguous. Considering that consultants may differ in their ability, we show that the optimal contracts enable senior doctors (with more experience) to get a greater private income than junior doctors when discrimination between senior and junior physicians is allowed. When discrimination is not allowed, it is optimal to offer a uniform contract. Proposing distinct contracts, as currently done in Ireland, increases healthcare costs due to incentive compatibility issues.

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1. Introduction

In some countries, access to medical care in public hospitals, as an outpatient, can be subject to extremely long waiting times. The issue is particularly stringent in Ireland and regularly documented in the press: "Over 140,000 are still believed to be on outpatient lists, with some people waiting up to eight years to see a consultant in an outpatient clinic" (Irish Health, 2008).

The costs associated with delays in accessing care and getting elective surgeries are generally substantial, but they are also difficult to estimate. Hurst and Ceciliani (2003) show that these vary greatly across conditions, across countries and through time. One reason these costs are difficult to assess stems from the fact that they can include, among other factors, the deterioration in the condition of the patient, the loss of utility suffered by the patients, an increase in the cost of surgery and of other treatments, and potential loss of income (see Naylor et al. (1994) and Harrison and New (2000)).

To access care within a shorter period, some patients may be able to avail private care either in a private hospital or within the public hospital. In some countries, physicians employed in a public hospital are permitted to treat private patients within the public hospital. According to Paris et al. (2010) such dual practice is authorized in 16 out of the 29 OECD member states.[2]

Recently, the privilege to engage in such dual practice has been under scrutiny as it has led to concerns about unequal time allocation on behalf of consultants and, consequently, unequal access to health care.[3] In Ireland, a special commission working on behalf of the government (Sláintecare) concluded that "disentangling and removing private care from public hospitals is essential to delivering universal healthcare in Ireland." (The Irish Times, 2018). In response, The Irish Medical Organisation (the professional association for medical doctors in Ireland) argued that the removal of private care from public hospitals may make the waiting list problem worse if it leads consultants to quit their jobs in public hospitals to join private hospitals (Irish Medical Organisation, 2018). García-Prado and González (2007) list different countries in which consultants can treat private patients in public hospitals and raise concerns that this may adversely affect competition for private treatments.

From an economic perspective, and more particularly from a contract theory perspective, allowing for private practice within public hospitals provides important advantages and it is not clear whether it is sub-optimal. Firstly, and as argued in García-Prado and González (2007), it enables the hospital manager to verify and control the

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[2] In Ireland, the provision of private care in public hospitals was formalized in 1997 as it became part of the physicians' contracts (Health Service Executive, 1997): Consultants who are hired in a public hospital can either opt for a contract which allows them to treat public patients only or they can select a contract allowing them to provide private care to no more than 20% of their outpatients.

[3] Freed et al. (2017) provide an analysis of how consultants allocate their time between private and public patients in Australia. González et al. (2017) use data from Indonesia to show that dual practice increases the number of private patients seen.

actual number of private outpatients that are treated by a consultant. Secondly, private outpatients provide an additional income (via a fee for service) to the consultants and, potentially, to the hospital. This additional monetary incentive can however influence the consultant's incentives and shift their focus on seeing as many patients as possible increasing the risk of misdiagnosis.

This paper proposes a theoretical analysis of the private provision of care in public hospitals and assesses its impact on the quality and cost of healthcare. Keeping in mind the waiting list issue, we capture this policy's impact on the number of outpatients that are seen and on the number that are cured, considering that one may differ from the other depending on the level of care exerted by consultants.

The analysis relies on the supply-side of health care for outpatients. The demand side is taken as given and composed of numerous outpatients which are all identical in terms of the severity of their illness. We consider that there is excess demand for public consultations. The demand for private consultations depends, negatively, on the level of the private fee for service.[4] The optimality of dual practice hinges depends on the revenue gathered from the private practice which is endogenously determined.

We consider that consultants respond to monetary incentives and that they value patients' well-being. This is captured assuming that consultants benefit from curing patients via some intrinsic motivation. They get an equal increase in utility from curing a public or private patient, but also value the fee for service paid by the private patient.

Consultants bear a cost of providing care and have a unique expertise that enables them to decide on the total number of outpatients they can attend to. Importantly, we introduce a distinction between the number of patients being seen and the number being cured to capture the potential risk that may result from a misdiagnosis as more patients are seen. These two numbers depend on the level of care chosen by the doctor. The number of patients seen is decreasing with the level of care being granted to each patient. The number of patients being cured is inverse-U shaped with respect to the level of care. When exerting low levels of care, the consultant can see many patients but cures few. When exerting high level of cares, more patients are cured but fewer are seen.

The objective of the hospital manager is to implement contracts that maximize the number of outpatients being cured while minimizing health expenditures. The monetary value associated with each cured outpatient can be understood as reflecting the cost that the hospital manager must cover should this patient remain unwell. In other words, we introduce a parameter that captures the cost associated with delays due to waiting lists and characterize the optimal contract for all possible values of this parameter.

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[4] In practice, this fee is at times covered (entirely or partially) by the patient's health insurance company. However, the private insurance policies which cover private consultations tend to be more expensive. Also, the coverage is typically capped. It is not unreasonable to assume that insurance companies would cover fewer private consultations as the fee per consultation increases.

The contracts the hospital manager offers specify the fixed salary that consultants receive as well as the percentage of private outpatients they can attend to and the resulting private income it would generate.

We establish the following results. The additional private revenue gathered from setting up private clinics in public hospitals has a positive, first order effect, on the wages. Every cent gathered from the private clinic is a cent that the hospital manager can save. Increasing the private per patient revenue has a negative second order effect on the hospital's profits as it decreases the number of patients being cured. Therefore, when the focus is on reducing health expenses, it is optimal to set the percentage of private patients so that the revenue doctors get from private care is maximized.

As the cost associated with waiting lists increases and the focus of the hospital manager shifts towards curing patients, the manager selects a percentage of private patients such that the revenue from the private clinic decreases. This leads consultants to shift their attention to curing more patients. The optimal percentage is not always unique as the same revenue can be generated via a low percentage of private outpatients and a high fee or a larger number of private patients and a lower fee. The complete removal of private practice is optimal when costs associated with having patients remaining on waiting lists is very high.

As we allow for heterogenous consultants, who differ in their ability, our main conclusions are the following. When the hospital manager can use seniority in order to discriminate across consultants, and when senior consultants have a greater ability than junior doctors, we show that senior consultants are the ones who should benefit the most from the private clinics. In other words, their income from private consultations is no less than what junior doctors get from the private clinics. Consultants with a greater ability have a greater intrinsic motivation and provide higher levels of care. The removal of private practice aims at leading doctors to focus more on care. There is therefore less of a need to remove dual practice in the presence of doctors who are naturally inclined to provide more care. Whether both types of doctors are allowed seeing private patients depends on the cost associated with delays.

When discrimination is not possible, we show that offering separating contracts increases the cost of healthcare because it increases the cost associated with incentive compatibility and therefore it is not optimal. Under the optimal uniform contract, the more experienced consultants receive some informational rents, but these are solely based on their greater returns from curing patients and are not associated with any financial burden for the hospital.

In conclusion, and in relation to the current debate on the optimality of the private care in public hospitals, we find that (i) it does lead to an adverse effect on the level of care being exerted as consultants focus more on numbers of patients seen and that (ii) it reduces the cost associated with the overall provision of care. Eliminating private care in

public hospitals will lead consultants for focus more on the number of patients being cured and they will see less patients. Hence the impact on waiting lists is not trivial.

The next section provides a review of related papers. Section three describes the model. In section four we analyse the optimal contract when consultants are identical. We extend the analysis to heterogeneous consultants in section five. Finally, we conclude in section six.

2. Literature Review

The issue we analyse is relevant as several countries authorize private clinics within the public hospitals. As mentioned above, 16 OECD member states allow consultants to see private patients within the public hospital. Paris et al. (2010) explain that, in some countries (such as Belgium) this policy results from the fact that consultants are self-employed. In other countries such France, Ireland and the UK, consultants are employees of the public hospital and their contract stipulates that they can treat some patients privately. In France, it is suggested that this privilege was introduced to retain highly qualified consultants. In most instances the revenue gathered from the private provision of care is limited. In the UK doctors with a full-time contract from English National Health Service (NHS), have a private income limited to 10% of their NHS salary (Raffel, 2007). In France, consultants who engage in private practice cannot earn a private income that is above 30% of their overall income (Kiwanuka et al., 2011). In Ireland, a consultant can see at most 20 private patients out of 100 (Health Service Executive, 2019).

In this paper we analyse specifically contracts that allow consultants to treat private patients within a public hospital. In general, dual practice can also take the form of allowing consultants to split their time between a public hospital and a separate, private clinic or hospital. Such a possibility raises further issues and several papers focus on the costs and benefits of dual practice agreements (see Bir and Eggleston (2006)).

When it comes to incentives and their relation to the provision of private health care it is not straightforward to reach a consensus. Medical doctors typically respond to extrinsic, monetary incentives but they also tend to be altruistic and have reputational concerns. Here we show that the establishment of private clinics within public hospitals lead consultants to shift their focus on seeing more patients and thereby lowering the amount of care each receives. Brekke and Sørgard (2006) provide further arguments supporting the fact that dual practice can lead to an overall deterioration of health care provision. Biglaiser and Ma (2007) and Delfgaauw (2007) argue that a less pessimistic outcome can arise when accounting for the presence of altruistic consultants. Indeed, the argument is based on the possibility that the private sector may appeal to doctors who are less altruistic and respond more to monetary incentives. These get to exert higher levels of care in the private sector, leaving the public sector filled with devoted doctors. González (2004) refers to a doctor's reputational concerns and show that consultants can

provide high quality care in a public hospital to promote his prestige as a private practitioner. These analyses assume that the public and private sector are separate entities, and do not address the private provision of care within a public hospital.

The consensus that emerges from the literature is the need for regulation when dual practice is introduced. García-Prado and González (2007) provide an extensive review of the different policies that are used in different countries and highlight their associated benefits and risks. Within this literature González and Macho-Stadler (2012) provides a theoretical comparison of distinct regulatory measures. Interestingly, these authors show that limiting consultant's involvement in dual practice is more effective than limiting earnings from the private practice. This is a subtlety our model fails to capture as we link the fee for service to the number of outpatients that seek private treatment. However, in their approach, the private provision of care is exerted outside the public hospital.

Finally, the decision to ban dual practice in public hospitals may lead some consultants to leave and opt for a position in a private hospital. One concern may then be that public hospitals may end up with less qualified consultants who do not have outside options in the private sector. Barigozzi and Burani (2016) provide evidence to the contrary as they analyse specifically the allocation of medical doctors between a public and a private hospital and point to an efficient sorting of consultants.

3. The Model

The patients: We consider that there are numerous patients all identical in the severity of their illness. The ones who attend the hospital as public outpatients do not pay for the healthcare they receive. Those who attend the hospital as private outpatients must pay a fee for service s which depends on the percentage of private patients that can be attended to by a consultant: $\alpha \in [0,1]$. Specifically, we assume that $s(\alpha)$ is a decreasing function of α. This is motivated assuming that demand for private health provision decreases as the fee for such services increases. We assume that $s(1) \geq 0$.

The hospital manager: We consider a public hospital manager who must contract medical consultants. The contracts she can offer specifies the wage that medical practitioner i receives (w_i), as well as the percentage of private patients he can attend to (α_i). The manager's objective consists in maximizing the number of patients being cured while minimizing expenses.

Let C_i denote the number of patients cured by consultant i. We assume that each cured patient has a monetary value of $\theta \geq 1$. This value could reflect the benefit of treating a

patient who would otherwise remain ill. The longer the waiting lists are, the more delays the outpatients experience and the larger θ is.

The manager's objective consists in maximizing

$$\Pi = \sum_{i=1,\dots,I} \theta C_i - w_i.\text{[5]}$$

The consultants

Consultants are experts in their discipline and the hospital manager recognises this expertise. Therefore, consultants can decide on the number of patients they can attend to and, consequently, the amount of care he gives to each patient. This decision results from balancing different concerns. On the one hand doctors are interested in financial returns which would incentivise them to see a larger number of private patients. On the other hand, they care about the patients' wellbeing and this means that they need to find the right treatment for each patient.

Specifically, we let e denote the care (or effort) that a consultant devotes to a patient. The number of patients that are seen is then given by $N(e) = 1 - \rho e$ with $\frac{1}{2} < \rho < 1$. The number of patients that cured is given by $C(e) = eN(e)$. Notice that the number of patients being cured is maximized when $e = \frac{1}{2\rho}$, in which case $N = \frac{1}{2}$ patients are seen while $C = \frac{1}{4\rho}$ are cured.[6]

A practitioner's utility function is then given by $u = w + v(e, \alpha)$, where

$$v(e, \alpha) = \gamma C(e) + s(\alpha)\alpha N(e) - \frac{1}{2}e^2.$$

The last term measures the cost of providing care. Curing patients, as opposed to simply seeing them, triggers an increase in utility captured via the parameter γ which reflects a consultant's ability. Finally, the middle term is the private income that a consultant gathers from exerting dual practice. Hence, this utility captures the fact that consultants respond to monetary incentives as well as altruistic concerns.

Let $r(\alpha) = \alpha s(\alpha)$ denote the per-patient revenue from the provision of private practice. It reaches a maximum at α^* such that $r'(\alpha^*) = 0$.[7] We make the following assumption concerning the function $r(.)$.

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[5] Below, we discuss the possibility that the hospital retains a revenue $\tau \sum_i \alpha_i N_i$ from the private patients.
[6] Although care is beneficial to a patient, we incorporate the fact that a physician can exert too much care as in Woodward and Warren (1984).
[7] We use prime to denote first derivative.

Assumption: The function $r(\alpha) = \alpha s(\alpha)$ is such that $r(1) \geq 0$. Under this assumption, and for any $\alpha_2 \in]\alpha^*, 1]$ there always exists a unique $\alpha_1 \in]0, \alpha^*[$ such that $r(\alpha_1) = r(\alpha_2)$. Moreover, there exists $\underline{\alpha} \geq 0$ such that $r(\underline{\alpha}) = r(1)$ and $r(\alpha) < r(1)$ for any $\alpha < \underline{\alpha}$.

Figure 1, below, gives an illustration of a per-patient revenue that satisfies the assumption made above.
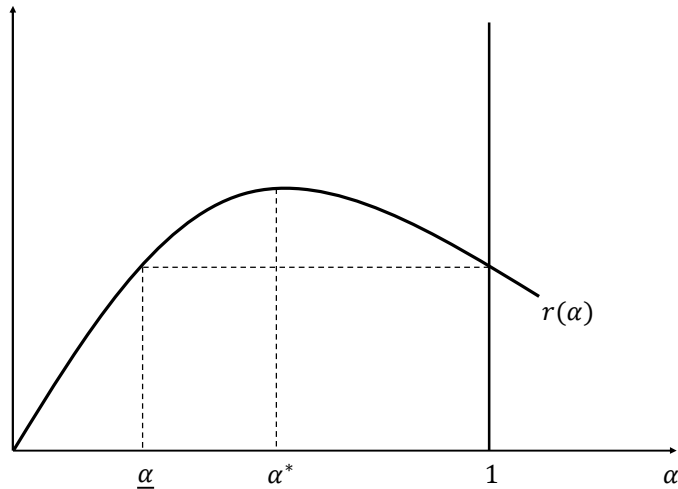

Figure 1: Per-patient private revenue.

Finally, consultants have the same reservation utility capturing the level of utility they would get accepting their next best offer, it is denoted by $\underline{u}$.

The timing of the game is as follows. First the hospital manager issues the contracts to the consultants she wishes to hire. If there is a unique contract or if discrimination is possible, doctors decide whether to accept or reject the contract they are offered. If there are several contracts and discrimination is not allowed, each consultant is free to choose the contract that suits him best. Once they accept the contract, each doctor decides on the level of care he wishes to devote to his patients. In other words, each tells the hospital manager how many patients he can attend to.

The game we are considering is a sequential game with complete information and we solve for the subgame perfect equilibrium. We therefore solve the game by backward induction.

4. Optimal contracting with homogeneous consultants

Assume that all consultants are identical in their ability. In this case, to solve for the optimal contract we can consider that the hospital manager faces a single, representative physician for whom she must design an optimal contract perfectly anticipating the level of care he exerts.

- **Optimal level of care and the number of patients cured.**

Given any contract (w, α), the consultant chooses a level of care such that $\frac{\partial v}{\partial e} = 0$.[8] This leads to

$$e^* = \frac{\gamma - \rho r(\alpha)}{1 + 2\rho\gamma}. \tag{1}$$

Note that the level of care exerted responds to the percentage of private patients only via its impact on the private revenue $r(\alpha)$. For any $\alpha_1 < \alpha_2$ such that $r(\alpha_1) = r(\alpha_2)$ the same level of care is exerted.

The level of care decreases with $r(\alpha)$ suggesting that allowing consultants to operate a private clinic in the public hospital will adversely affect the level of care that they provide because a private income shifts the attention of consultants on the number of outpatients seen.

Let e^c denote the level of care that would maximize the number of patients being cured $(1 - 2\rho e_c = 0)$. We have

$$\left.\frac{dv}{de}\right|_{e=e^c} = -\rho r(\alpha) - e^c < 0. \tag{2}$$

Therefore, $e^* < e^c$: the consultant devotes too little care to the patients to maximize the number of patients being cured. This is because he bears the cost of providing the care and because the provision of private care incentivises the consultant to see, but not necessarily cure, more patients.

As Figure 2 below gives a visual illustration of the level of care and its relation to e^c. The main points this figure captures are the following. Firstly, note that the level of care decreases as the private per-patient revenue $r(\alpha)$ increases. Hence, allowing for the private provision of health depresses the level of care exerted by consultants because they are incentivised to see more patients. Secondly, note that all possible e^* are such that $C'(e) > 0$. As argued above, the number of cured patients is sub-optimal.
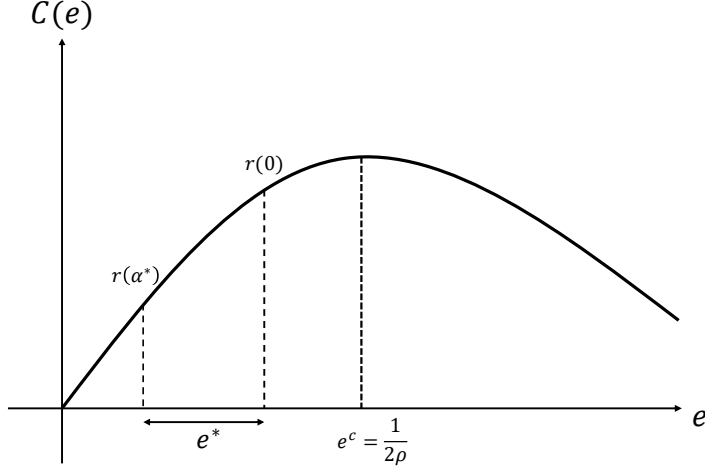
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[8] The function $v(.)$ is concave in effort.

Figure 2: Representing $C(e)$ and the range of exerted care e^*.

- **Optimal contracts and provision of private care.**

The hospital perfectly anticipates the decision of consultant and selects a contract that maximizes

$$\Pi = \theta e^* N(e^*) - w,$$

subject to

$$w + v(e^*, \alpha) \geq \underline{u},$$

where $\underline{u}$ denotes the consultant's reservation utility and where e^* is given by (1).

Clearly, the hospital will set the wage such that the reservation utility binds, and it maximizes

$$\Pi(\alpha) = \theta e^* N(e^*) + v(e^*, \alpha) - \underline{u}.$$

Since $\frac{\partial v}{\partial \alpha} = r'(\alpha)N(e^*)$, the variable α affects profits via $r(\alpha)$ which, itself, characterizes the optimal care level, e^*. Hence, the optimal policy is one that should possibly cap the income the consultant gathers from the private clinic. In that respect, the hospital could be indifferent between a high or low percentage of private patients which yield the same private per-patient revenue. In other words, the expected profits would be the same for any α_1 and α_2 such that $\alpha_1 < \alpha^* < \alpha_2$ and $r(\alpha_1) = r(\alpha_2)$.

Let

$$\theta_l = \frac{(1 + 2\rho\gamma)(1 + \rho\gamma + \rho^2 r^*)}{\rho(1 + \rho^2 r^*)},$$

where $r^* \equiv r(\alpha^*)$ and let

$$\theta_h = \frac{1 - \gamma\beta}{\beta(1 - 2\gamma\beta)}.$$

One can show that $\theta_l < \theta_h$ (see Appendix).

Proposition 1: *For all $\theta \leq \theta_l$, the optimal value for α is α^* and the per-patient revenue from the private clinic is maximized. This leads to the lowest amount of care being exerted and a maximum number of patients being seen.*

For all θ such that $\theta_l < \theta < \theta_h$ the optimal percentage of private patients is such that the per-patient private revenue is set below the optimal level: $r < r(\alpha^*)$. This means that a greater amount of care is exerted but less patients are seen. The optimal r is unique. In some cases, there are two optimal values of α, each associated with the same $r(\alpha)$.

Finally, for any $\theta \geq \theta_h$, the optimal value for α is $\alpha = 0$ and the highest amount of care is exerted meaning that the higher number of patients are cured.

Proof: see Appendix.

We now give a short intuition for the results above. Let $g(e) = (1 - \rho e) - \beta\theta(1 - 2\rho e)$, where $\beta = \frac{\rho}{1 + 2\rho\gamma}$. Taking the first derivative of the objective function we get

$$\frac{d\Pi}{d\alpha} = g(e^*)r'(\alpha) = 0.$$

where e^* given by (1).

The hospital must select the optimal percentage of private patients that maximizes the number of patients being cured and minimizes expenses. When the focus is more on the expenses, so that θ is low, it is optimal to set the per-patient revenue as high as possible since a penny received from a private patient is a penny saved for the hospital. This solution is the one that arises when $\theta \leq \theta_l$ and we have $\alpha = \alpha^*$.

When the focus shifts towards patients being cured, the optimal percentage of private patients is set such that $g(e^*) = 0$ where e^* is the level of care chosen by the consultant. In such cases we potentially have two optimal values for α (each associated with the same $r(\alpha)$).

Finally, as θ becomes very large, then it is optimal to maximizes the number of patients being cured which we achieve by setting $\alpha = 0$.

Illustrative example: Assume that $N(e) = 1 - 0.8e$, $\gamma = 1$, and let $s(\alpha) = 1 - 0.8(\alpha)$ so that $\alpha^* = 62.5\%$.

For $\theta \in [1, 4.65]$ the optimal solution is to set $\alpha = \alpha^*$. For $\theta \in]4.65, 4.95]$ the optimal solution is not unique, and we have two values, on either side of α^* that lead to the same expected profit. At $\theta = 5$, setting $\alpha = 100\%$ and $\alpha = 25\%$ leads to the same profits for the hospital.

For $\theta \in]4.95, 5.74]$ the optimal solution is unique and the optimal α is decreasing in θ. For $\theta \geq 5.74$, the hospital manager ceases to offer any private clinics and sets $\alpha = 0$.

Figure 3 below illustrates the solution.
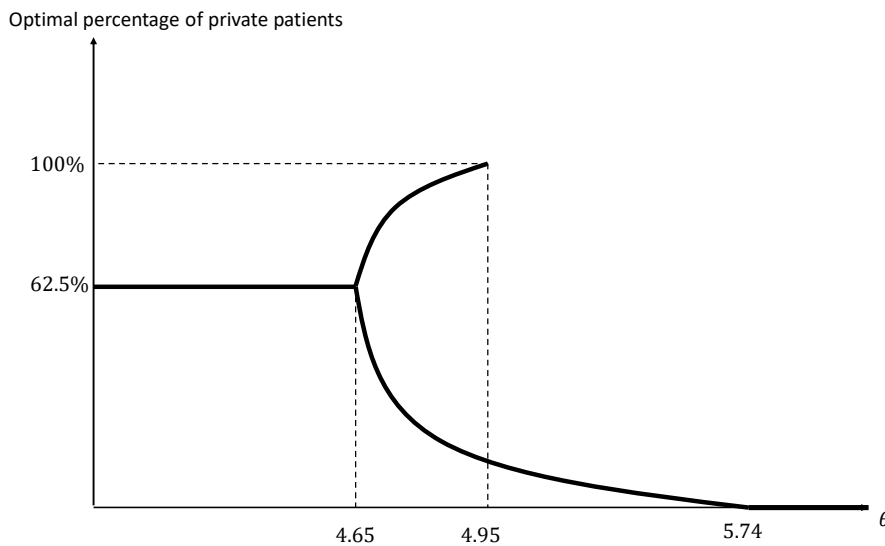

Figure 3: Optimal percentage of private patients when $N(e) = 1 - 0.8e$, $\gamma = 1$, and $s(\alpha) = 1 - 0.8(\alpha)$.

- **Optimal contracts when private patients are a source of private revenue for the hospital.**

Before we extend the analysis and allow for heterogenous consultants, we consider what happens when the hospital receives a private revenue from private patients. That would be the case if part of the fee paid by the patients who choose to attend the private clinic went to the hospital. Let us assume that

$$\Pi = e^* N(e^*) + \tau \alpha N(e^*) - w,$$

where the second term is the private revenue gathered by the hospital and e^* is given by (1).

In such a case we have

$$\frac{d\Pi}{d\alpha} = [g(e^*) + \alpha\beta\rho\tau]r'(\alpha) + \tau N(e^*) = 0.$$

Any interior solution that prevails when $\tau = 0$ is such that $g(e^*) \geq 0$ (see Appendix). Therefore, for any $\theta \leq \theta_h$, the optimal value for α is greater than α^* as we must have $r'(\alpha) < 0$.

Clearly, when the hospital gets part of the private fee for service that is paid by private outpatients, it has an incentive to allow more patients to be seen privately. More importantly, the optimal percentage of private patients matters in its own right and not only via its role in determining the consultant's private income.

5. Optimal contracting with heterogeneous consultants

In many countries public hospitals' managers offer distinct contracts to the consultants they employ. In Ireland specifically, two contracts are proposed to consultants but only one of these contracts allows consultants to treat private patients within the public hospitals.[9] The only rationale for offering different contracts is that consultants must be heterogeneous.

In this section we assume that consultants differ in relation to their ability. Specifically, we consider two types of consultants: A and B. We consider that type A consultants have a better ability, or experience, than type B consultants. We capture this assuming that, for a given effort level, type A consultants can cure more outpatients. In other words, we introduce a distinction assuming that $\gamma_A > \gamma_B$.[10]

While experience or ability can be verifiable, the legislation in a specific country may or may not allow hospital managers to discriminate across consultants. We will therefore consider a situation in which discrimination is possible and one where it is not.

5.1 Optimal contracts under discrimination

In the UK, seniority serves as a base for discrimination. Contracts offered to senior physicians can differ from those offered to junior consultants. Let us assume that type A consultants are senior consultants who have accumulated knowledge and experience and are able to cure more patients for a given level of effort. The question we address here is which, of the senior and junior consultants, can treat private patients?

In section 4 we characterized the optimal contracts when consultants are homogeneous. We can use these results as the hospital manager would treat each consultant individually

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[9] To be precise, there is a third type of contracts that is offered to very few consultants allowing them to practice in a private clinic or hospital.
[10] Alternatively, the parameter γ could reflect the level of altruism and be private information. Jack (2005) provides an analysis of optimal contracting under such asymmetric information.

when discrimination is possible and offer, to each, the optimal contract defined in Proposition 1, tailored to his type. Specifically, we found that consultants can treat private patients within the public hospital when $\theta \leq \theta_h$ where

$$\theta_h = \frac{1 - \gamma\beta}{\beta(1 - 2\gamma\beta)}.$$

Furthermore, the percentage of private patients is such that the revenue from the private clinic is maximized when $\theta \leq \theta_l$ where

$$\theta_l = \frac{(1 + 2\rho\gamma)(1 + \rho\gamma + \rho^2 r^*)}{\rho(1 + \rho^2 r^*)},$$

where $r^* \equiv r(\alpha^*)$.

The threshold θ_l and θ_h are increasing in γ so that $\theta_j(\gamma_B) < \theta_j(\gamma_A)$ ($j = l, h$).

Figure 4 gives a visual representation of the two possibilities that can arise according to whether $\theta_l(\gamma_A) < \theta_h(\gamma_B)$ or $\theta_h(\gamma_B) < \theta_l(\gamma_A)$.
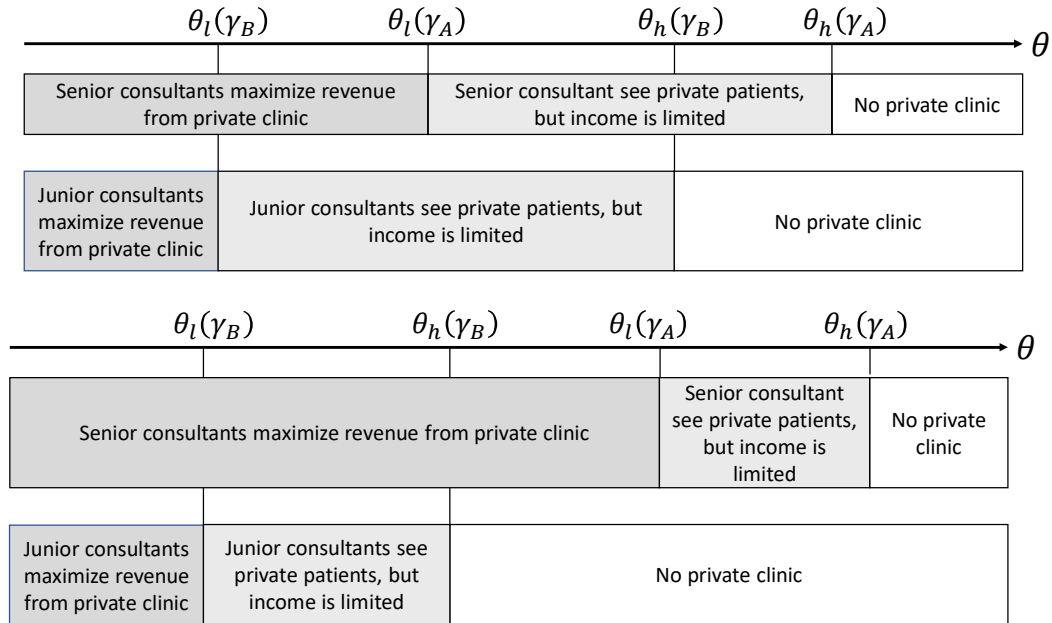

Figure 4: Optimal contracting terms for senior and junior doctors as a function of θ.

In the upper graph, we have $\theta_l(\gamma_A) < \theta_h(\gamma_B)$ and in the lower one we have $\theta_h(\gamma_B) < \theta_l(\gamma_A)$. On top, we specify the optimal contract for senior consultants. The lower tables specify the optimal contract for the junior consultants.

Clearly, in a setting where seniority can be used as a basis for discrimination, senior consultants get a revenue from attending private patients that is at least as great as the junior consultants' private revenue. In either case, there is a non-empty range for the parameter θ for which only senior consultants are allowed treating private patients. In either case, this outcome stems from the fact that senior consultants are more inclined to provide care and cure more patients. Hence there is a lesser need to impede them from exerting dual practice as this strategy serves as an incentive to shift the physician's focus on curing patients.

Interestingly, this finding may corroborate the fact that in France, only well-established senior consultants are allowed treating private patients according to Paris et al. (2010). The suggestion they make is not related to the level of care these provide but that this may be used to attract and retain such consultants.

5.2 Optimal contracts when discrimination is not feasible or illegal.

In Ireland, physicians joining any public hospital are offered two possible contracts and they can choose the one that they prefer. Therefore, the hospital manager's task consists in designing self-selective contracts. To find these contracts we must solve for a constrained optimisation problem which takes into consideration incentive compatibility.

Let by $u_i = w_i + v_i(e, \alpha_i)$, where $i = A, B$

$$v_i(e, \alpha_i) = \gamma_i C(e) + r(\alpha_i) N(e) - \frac{1}{2} e^2.$$

In equilibrium, the following efforts are exerted by the consultants

$$e_i^* = \frac{\gamma_i - \rho r(\alpha_i)}{1 + 2\rho\gamma_i}, i = A, B. \tag{3}$$

One can easily verify that, for a given $\alpha_A = \alpha_B$, $e_A^* > e_B^*$. As one would expect, the consultants with the highest ability devotes a higher amount of care, everything else being equal.

If, however, type i takes the contract aimed at type $j \neq i$, the following, off-equilibrium, effort level is exerted by type i:

$$e_{ij}^* = \frac{\gamma_i - \rho r(\alpha_j)}{1 + 2\rho\gamma_i}, i, j = A, B \text{ and } i \neq j. \tag{4}$$

The optimisation problem of the hospital, when it offers two contracts, is as follows.

$$\max_{(w_i,\alpha_i)_{i=A,B}} \sum_{i=A,B} e_i^* N(e_i^*) - w_i$$

subject to

$$w_i + v_i(e_i^*, \alpha_i) \geq \underline{u} \ (i = A, B),$$

$$w_i + v_i(e_i^*, \alpha_i) \geq w_j + v_i\left(e_{ij}^*, \alpha_j\right) \ (i, j = A, B \text{ and } i \neq j),$$

where

$$v_i\left(e_{ij}^*, \alpha_j\right) = \gamma_i C\left(e_{ij}^*\right) + r\left(\alpha_j\right) N\left(e_{ij}^*\right) - \frac{1}{2}\left(e_{ij}^*\right)^2.$$

The first constraints are the participation constraints while the last two are the incentive constraints. Notice that the incentive constraint for type A and the participation constraint for type B imply that the participation constraint for type A holds.

Indeed, note that we can re-write

$$w_A + v_A(e_A^*, \alpha_A) \geq w_B + v_A(e_{AB}^*, \alpha_B)$$

as

$$w_A + v_A(e_A^*, \alpha_A) \geq u_B + v_A(e_{AB}^*, \alpha_B) - v_B(e_B^*, \alpha_B).$$

Furthermore, we have

$$v_A(e_{AB}^*, \alpha_B) \geq v_A(e_B^*, \alpha_B) > v_B(e_B^*, \alpha_B),$$

Where the first inequality follows from the fact that $e_{AB}^* = \arg\max v_A(e_{AB}^*, \alpha_B)$ and the second inequality follows from the fact that $\frac{\partial v}{\partial \gamma} > 0$.

Hence, the only participation constraint that is relevant provides the optimal wage for type B as it is optimal to set w_B such that $w_B + v_B(e_B^*, \alpha_B) = \underline{u}$.

Lemma 1: *To guarantee that contracts are incentive compatible, the per patient revenue from the private practice for the less experienced consultant must be at least as great as the one gathered by the more experienced consultant: $r(\alpha_B) \geq r(\alpha_A)$.*

Proof: The incentive constraints can be re-written as follows:

$$IC_A: \ w_A - w_B \geq \left(r(\alpha_B) - r(\alpha_A)\right) h(\gamma_A),$$

$$IC_B: \ w_A - w_B \leq \left(r(\alpha_B) - r(\alpha_A)\right) h(\gamma_B),$$

Where

$$h(\gamma) = \frac{2(1 + \rho\gamma) + \rho^2\left(r(\alpha_B) + r(\alpha_A)\right)}{2(1 + 2\rho\gamma)}.$$

We have $\frac{\partial h}{\partial \gamma} < 0$ so that $h(\gamma_A) < h(\gamma_B)$. Therefore, we must have $(r(\alpha_B) - r(\alpha_A)) \geq 0$ for there to exist an interval for $(w_A - w_B)$ where both incentive constraints can hold.∎

Proposition 2: *The optimal contract is uniform: $w_A = w_B$ and $\alpha_A = \alpha_B$. The wage is set such that type B consultant gets exactly his reservation utility. This means that type A consultants get some informational rents due to their higher ability.*

Proof: See Appendix.

To understand the intuition behind these results, one must focus on the strength and weaknesses of each type of consultant. For any given contract, type A consultants would exert more care and thus see fewer patients than type B consultants. The reward type A consultants get comes from their ability to cure more patients. Type B consultants, by opposition, get a greater financial reward seeing more patients as they exert a lower level of care.

Hence, when discriminations based on ability is allowed, the hospital manager pays type A consultants a lower fixed wage due to their greater focus on an intrinsic reward. When such a discrimination is not allowed, constraint IC_A stipulates that for them to accept their contract, their fixed wage must be large enough or else they would take the contract aimed at type B consultants.

Setting a large enough fixed wage for type A consultants makes their contract more attractive to type B consultants. If, in addition to guaranteeing a fixed wage large enough the private fees were set such that $r(\alpha_A) > r(\alpha_B)$, type B consultants would opt for the contract aimed at type A consultants. Indeed, since they see more patients, they would get a greater income out of the private clinic.

Separation of types can be achieved by offering type B consultants a contract with a focus on the private clinic and by giving type A consultants a fixed wage that results from trading-off the need to keep the fixed wage high enough while profiting from their greater intrinsic motivation.

Setting $r(\alpha_A) = r(\alpha_B)$ eliminates the cost associated with the incentive constraints. It does not eliminate the informational rents that accrue to type A consultants. However, these rents are the outcome of their greater ability to cure patients. They compensate the consultant via an intrinsic appreciation of their work. In other words, these rents are not a source of any additional financial burden for the hospital.

An important conclusion, from proposition 2, is that the current practice, in Ireland, which consists in offering separating contracts increases the cost of healthcare because it increases the cost associated with incentive compatibility and therefore it is not optimal.

Knowing that the optimal contract is the same for all consultants, we now characterize the optimal amount of private care that consultants can provide.

Let

$$\theta_L = \frac{2\beta_B(1 + \rho\gamma_B + \rho^2 r(\alpha^*))}{(\beta_A^2 + \beta_B^2)(1 + \rho^2 r(\alpha^*))},$$

and

$$\theta_H = \frac{2\beta_B(1 + \rho\gamma_B)}{(\beta_A^2 + \beta_B^2)}.$$

One can show that $\theta_L < \theta_H$.

Proposition 3: *For all $\theta \leq \theta_L$, the optimal value for α is α^* meaning that the per-patient revenue from the private clinic is maximized. This leads to the lowest amount of care being exerted and a maximum number of patients being seen.*

For all θ such that $\theta_L < \theta < \theta_H$ the optimal percentage of private patients is such that the per-patient private revenue is set below the optimal level: $r < r(\alpha^*)$. This means that a greater amount of care is exerted but less patients are seen. Specifically, the manager selects α such that $g(e^*) = 0$. The solution may not be unique.

Finally, for any $\theta \geq \theta_H$, the optimal value for α is $\alpha = 0$ and the highest amount of care is exerted meaning that the higher number of patients are cured.

Proof: see Appendix.

The intuition for the proposition above is similar to that of proposition 1.

We can use Propositions 1 and 3 to make one last comparison and answer one last question: is it better to hire consultants with a similar profile or is a mix of senior (experimented consultants) and junior (less experimented) consultants better when it comes to the level of care, assuming that discrimination is not possible?

Assume that $\gamma_A = \frac{3}{2}\gamma$ while $\gamma_B = \frac{1}{2}\gamma$. Figure 5, below, shows how the thresholds compare. It highlights the fact that hospital managers are keener to authorize dual practice when consultants exhibit less discrepancies in ability.
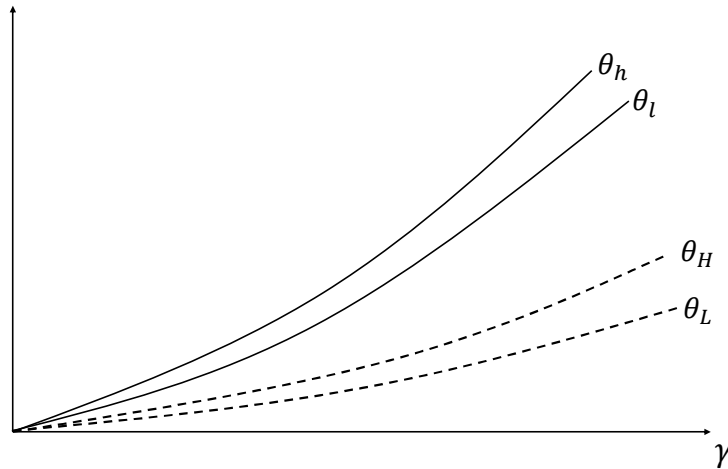
Figure 5: Representation of θ_h, θ_l, θ_H and θ_L for various values of γ, assuming that that $\gamma_A = \frac{3}{2}\gamma$ while $\gamma_B = \frac{1}{2}\gamma$.

In the presence of heterogenous consultants, the wage is such that consultants with the lowest ability get their reservation utility. Hence, heterogeneity across consultants puts a downward pressure on the wages. This, in turn, means that it is optimal to remove the private clinic (used to address the health care costs) for a wider range of θ.

Therefore, the level of care will be higher when the hospital manager hires heterogeneous consultants. Said differently, we find that the removal of private clinics within the public hospital will have a lesser negative impact in a situation where consultants are heterogenous.

6. Conclusions

This paper highlights the costs and benefits associated with the private provision of care within public hospitals which is under scrutiny at present in Ireland. It shows that allowing for private patients has a negative impact on the level of care being provided. It leads consultants to focus more on the number of patients seen rather than the number of patients cured. Whether it is optimal depends on the weight of health care costs relative to costs associated with waiting lists and delays in accessing care. The removal of private practice in public hospital is desirable when this cost is large, and the priority becomes the provision of care to cure many patients. That said, the removal of private clinic has an ambiguous impact on waiting lists as a greater dedication on behalf of consultants will lead to less patients being seen but more being cured.

Finally, we show that the current practice, in Ireland, which consists in offering separating contracts is optimal when discrimination is allowed. In such a case, senior consultants should receive a greater income from the private practice because they are naturally more inclined to provide care. When discrimination is not legal, offering distinct contracts (as currently done in Ireland) increases the cost of healthcare because it increases the cost associated with the incentive compatibility constraints and therefore it is not optimal.

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APPENDIX

Proof of Proposition 1: Let $\beta = \frac{\rho}{1+2\rho\gamma}$, so that we have $\frac{de}{dr} = -\beta$. Let the function $g(e)$ be defined as

$$g(e,\theta) = (1 - \rho e) - \beta\theta(1 - 2\rho e).$$

The first order condition can be written as

$$\frac{d\Pi}{d\alpha} = g(e^*)r'(\alpha) = 0,$$

where $r'(\alpha) = \frac{dr(\alpha)}{d\alpha}$ and e^* is given by (1).

The second order condition is given by

$$\frac{d^2\Pi}{d\alpha^2} = g(e^*,\theta)r''(\alpha) - \beta\big(r'(\alpha)\big)^2 \frac{dg}{de^*},$$

where $r''(\alpha) = \frac{d^2r(\alpha)}{d\alpha^2}$ and

$$\frac{dg}{de^*} = \rho(2\theta\beta - 1).$$

Any interior solution must satisfy the first and second order conditions: $g(e^*)r'(\alpha) = 0$ and $\frac{d^2\Pi}{d\alpha^2} < 0$ at the solution.

Solution for low values of θ

Let θ_l be defined such that $g(\underline{e},\theta_l) = 0$ where $\underline{e} \equiv e^*(\alpha^*)$ is the lowest level of care as α^* is defined such that $r'(\alpha^*) = 0$:

$$\theta_l = \frac{1 - \rho\underline{e}}{\beta\big(1 - 2\rho\underline{e}\big)}.$$

We have $g(\underline{e}) > 0 \Longleftrightarrow \theta < \theta_l$ since $g(e^*,\theta)$ is decreasing in θ.[11]

For any $\theta \leq \theta_l$ setting $\alpha = \alpha^*$ is optimal since it satisfies the first order condition and we have

$$\frac{d^2\Pi}{d\alpha^2} = g(\underline{e},\theta)r''(\alpha^*) \leq 0.$$

For any $\theta > \theta_l$ setting $\alpha = \alpha^*$ leads to a local minimum since $g(\underline{e},\theta) < 0$.

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[11] Recall that $C'(e^*) = (1 - 2\rho e^*) > 0$ for any α. See Figure 1 as an illustration.

Solutions for large values of θ

For any $\theta \geq \theta_l$, the function $g(e, \theta)$ is increasing in e.[12]

Let θ_h be defined such that such that $g(\overline{e}, \theta_h) = 0$, where $\overline{e} = e^*(0)$ is the highest level of care:

$$\theta_h = \frac{1 + \rho\gamma}{\beta}.$$

It is straightforward to verify that $\theta_l < \theta_h$.

For any $\theta > \theta_h$ then $g(\overline{e}, \theta) < 0$, meaning that $g(e^*, \theta) < 0$ for any α. Therefore, we have

$$\frac{d\Pi}{d\alpha} = g(e^*, \theta)r'(\alpha) \leq 0 \Leftrightarrow \alpha \leq \alpha^* \text{ and } \frac{d\Pi}{d\alpha} = g(e^*, \theta)r'(\alpha) \geq 0 \Leftrightarrow \alpha \geq \alpha^*.$$

Then the objective function is convex and decreasing in $r(\alpha)$ so that the optimal α is equal to zero.

Solutions for intermediate values of θ

Consider any $\theta \in]\theta_l, \theta_h[$. For any such values of θ we have, $g(\underline{e}, \theta) < g(e^*, \theta) \leq g(\overline{e}, \theta)$. Since $g(e, \theta)$ is increasing in e there exists $\hat{r}$ such that $g(e^*, \theta) = 0$ at $\hat{r}$ with $\underline{e} < e^*(\hat{r}) \leq \overline{e}$. It is optimal to select $r(\alpha) = \hat{r}$ as the first order condition is satisfied at this solution and we have

$$\frac{d^2\Pi}{d\alpha^2} = -\beta(r'(\alpha))^2 \frac{dg}{de^*} < 0.$$

Given that two distinct values of α can lead to the same $\hat{r}$ and the same effort, the solution in terms of α is not necessarily unique. However, in each case the value for $\hat{r}$ is the same and the solution is such that

$$e^*(\alpha) = \frac{1}{\rho}\left(\frac{\beta\theta - 1}{2\beta\theta - 1}\right) > 0.$$

One can easily verify that we have $\beta\theta - 1 > 0$ for any $\theta \geq \theta_l$.

We have two solutions so long as the lowest value of α solving $g(e^*(\hat{r}), \theta) = 0$ is greater than $\underline{\alpha}$.[13] When it is lower than $\underline{\alpha}$ the solution is unique and converges to 0 as θ increases.

---

[12] One can verify that $\theta_l > \frac{1}{2\beta}$, hence, for any $\theta \geq \theta_l$ we have $(2\theta\beta - 1) > 0$ so that $g'(e) > 0$.

[13] See Figure 1 in the text for a definition of $\underline{\alpha}$.

Proof of Proposition 2

The participation constraint for type A is not relevant as it would follow from the participation constraint of type B and type A's incentive constraint. Therefore, the optimisation problem, when it offers two contracts, can be written as follows.

$$\max_{(w_i,\alpha_i)_{i=A,B}} \theta\big(C(e_A) + C(e_B)\big) - w_A - w_B$$

subject to

$$PC_B: \ w_B + v_B(e_B^*, \alpha_B) \geq \underline{u}, \text{ where } e_B^* \text{is given by (3) in the text,}$$

$$IC_A: \ w_A - w_B \geq \big(r(\alpha_B) - r(\alpha_A)\big)h(\gamma_A),$$

$$IC_B: \ w_A - w_B \leq \big(r(\alpha_B) - r(\alpha_A)\big)h(\gamma_B),$$

$$\big(r(\alpha_B) - r(\alpha_A)\big) \geq 0.$$

In order to minimize wages, w_B is set such that the participation constraint binds so that IC_B, which provides an alternative lower bound for w_B is not relevant. By opposition, IC_A provides a lower bound for the type A wage. Hence, the objective of the hospital can be written as

$$\max_{(w_i,\alpha_i)_{i=A,B}} \theta\big(C(e_A) + C(e_B)\big) + 2v_B(e_B^*, \alpha_B) - \big(r(\alpha_B) - r(\alpha_A)\big)h(\gamma_A) - 2\underline{u}$$

subject to $\big(r(\alpha_B) - r(\alpha_A)\big) \geq 0$.

Let λ denote the Lagrange multiplier associated with the constraint:

$$\mathcal{L} = \theta\big(C(e_A) + C(e_B)\big) + 2v_B(e_B^*, \alpha_B) - \big(r(\alpha_B) - r(\alpha_A)\big)h(\gamma_A) - 2\underline{u} - \lambda\big(r(\alpha_B) - r(\alpha_A)\big)$$

Let

$$g_i(e) = (1 - \rho e) - \beta_i\theta(1 - 2\rho e) \text{ with } \beta_i = \frac{\rho}{1 + 2\rho\gamma_i}.$$

The first order conditions can be written as

$$r_A'[g_A(e_A^*) + \lambda] = 0$$

$$r_B'\big[g_B(e_B^*) + \rho(e_A^* - e_B^*) - \rho\beta_A\big(r(\alpha_B) - r(\alpha_A)\big) - \lambda\big] = 0$$

$$\lambda\big(r(\alpha_B) - r(\alpha_A)\big) = 0,$$

where $r_i' = \frac{dr(\alpha_i)}{d\alpha_i}$, and e_i^* $(i = A, B)$ are given by (3) in the text.

In the remaining of the proof, it is important to notice that

$$(1 - \rho e_i(r)) = \frac{1 + \rho\gamma_i + \rho^2 r}{1 + 2\rho\gamma_i} \quad \text{and} \quad (1 - 2\rho e_i) = \frac{1 + \rho^2 r}{1 + 2\rho\gamma_i}.$$

Inexistence of a separating equilibrium.

We prove that such an equilibrium does not exist by contraction.

Assume that there exists a separating equilibrium where $\alpha_A^* \neq \alpha_B^*$ and such that $r_B^* > r_A^*$, where $r_i^* = r(\alpha_i^*)$. In any such equilibrium we must have $r_A'(\alpha_A^*) \neq 0$ since the function $r(.)$ reaches a maximum at $r' = 0$ and therefore we could not have a solution where $r_B^* > r_A^*$.

Moreover, any separating equilibrium is such that $\lambda = 0$ to satisfy the first order condition.

Therefore, the only possible optimal values for α_A must be such that $g_A(e_A^*) = 0$.

Moreover, it is optimal to set α_B^* such that $r_B^* > r_A^*$ provided the derivative of the profits with respect to $r(\alpha_B)$ is non-negative at $r(\alpha_B) = r_A^*$:

$$g_B(e_B) + \rho(e_A^* - e_B) - \rho\beta_A(r(\alpha_B) - r_A^*) > 0 \text{ at } r(\alpha_B) = r_A^*.$$

That is, we must have

$$g_B(e_B) + \rho(e_A^* - e_B) > 0 \text{ at } r(\alpha_B) = r_A^*.$$

After some simplifications, the above can be re-written as

$$H(\gamma_B) = g_B(e_B) + \rho(1 - 2\rho e_A^*)\frac{(\gamma_A - \gamma_B)}{1 + 2\rho\gamma_B}.$$

Notice that when $\gamma_B = \gamma_A$ the expression above is equal to 0.

We then have

$$\frac{\partial H}{\partial \gamma_B} = \frac{\partial g_B}{\partial e_B}\frac{\partial e_B}{\partial \gamma_B} - \rho(1 - 2\rho e_A^*)\frac{(1 + 2\rho\gamma_A)}{(1 + 2\rho\gamma_B)^2}.$$

This leads us to

$$\frac{\partial H}{\partial \gamma_B} = \rho(2\theta\beta_B - 1)\frac{1 + \rho^2 r_A^*}{(1 + 2\rho\gamma_B)^2} - \rho(1 - 2\rho e_A^*)\frac{(1 + 2\rho\gamma_A)}{(1 + 2\rho\gamma_B)^2}$$

Using the fact that $(1 - 2\rho e_A^*) = \frac{1 + \rho^2 r_A^*}{1 + 2\rho\gamma_A}$, the sign of $\frac{\partial H}{\partial \gamma_B}$ is simply the sign of $(\theta\beta_B - 1)$.

Now, notice that in equilibrium, since $g_A(e_A^*) = 0$, we must have $(\theta\beta_A - 1) > 0$.

Indeed, we have $g(e, \theta) = 0 \Leftrightarrow \theta = \frac{(1 - \rho e)}{\beta(1 - 2\rho e)}$. One can then easily verify that

$$\frac{1}{\beta} < \frac{(1 - \rho e)}{\beta(1 - 2\rho e)}.$$

Therefore, if $\theta = \frac{(1 - \rho e)}{\beta(1 - 2\rho e)}$ it is such that $(1 - \theta\beta) < 0$.

Since $\beta_B > \beta_A$, we have $(\theta\beta_B - 1) > (\theta\beta_A - 1) > 0$. The function $H(\gamma_B)$ is increasing. Therefore, for any $\gamma_B < \gamma_A$, $H(\gamma_B) < H(\gamma_A) = 0$. This contradicts the fact that, in a separating equilibrium, we must have $H(\gamma_B) > 0.\blacksquare$

Proof of proposition 3

The proof below follows the same approach as the proof of Proposition 1.

From the previous appendix we know that the optimal contract is uniform. Therefore, we can maximize the following objective function

$$\max_{\alpha} \theta\big(C(e_A^*) + C(e_B^*)\big) + 2v_B(e_B^*, \alpha_B) - 2\underline{u}$$

Let $r'(\alpha) = \frac{dr(\alpha)}{d\alpha}, r''(\alpha) = \frac{d^2r(\alpha)}{d\alpha^2}$ and $\beta_i = \frac{\rho}{1+2\rho\gamma_i}$.

We define $G(e_A, e_B)$ as

$$G(e_A, e_B) = 2(1 - \rho e_B) - \theta\big(\beta_A(1 - 2\rho e_A) + \beta_B(1 - 2\rho e_B)\big).$$

The first order condition can be written as: $r'(\alpha)G(e_A^*, e_B^*) = 0$, where e_i^* is the optimal level of care chosen by type $i = A, B$.

The second order condition is given by

$$r''(\alpha)G(e_A^*, e_B^*) - (r'(\alpha))^2 \left[\beta_A \frac{\partial G}{\partial e_A^*} + \beta_B \frac{\partial G}{\partial e_B^*}\right] \le 0$$

at the solution.

Notice that

$$\frac{\partial G}{\partial e_A^*} = 2\rho\beta_A\theta > 0 \ \ and \ \ \frac{\partial G}{\partial e_B^*} = 2\rho(\beta_B\theta - 1).$$

Solution for low values of θ

Let θ_L be defined such that $G\big(\underline{e}_A, \underline{e}_B\big) = 0$ where $\underline{e}_i = e_i^*(\alpha^*)$ is the lowest amount of care as α^* is defined such that $r'(\alpha^*) = 0$:

We have $G\big(\underline{e}_A, \underline{e}_B\big) > 0 \Leftrightarrow \theta < \theta_L$ since $G\big(\underline{e}_A, \underline{e}_B\big)$ is decreasing in θ. Therefore, for any $\theta \le \theta_L$ setting $\alpha = \alpha^*$ is optimal since it satisfies the first order condition and we have

$$\frac{d^2\Pi}{d\alpha^2} = r''(\alpha)G\big(\underline{e}_A, \underline{e}_B\big) \le 0.$$

For any $\theta > \theta_L$ setting $\alpha = \alpha^*$ leads to a local minimum since $G\big(\underline{e}_A, \underline{e}_B\big) < 0$.

Solutions for large values of θ

For any $\theta \ge \theta_L$, the function $G(.)$ is increasing in e_A^* and e_B^*.

Let θ_H be such that $G(\bar{e}_A, \bar{e}_B) = 0$ where $\bar{e}_i = e_i^*(0)$ is the highest amount of care:

$$\theta_H = \frac{2\beta_B(1 + \rho\gamma_B)}{(\beta_A^2 + \beta_B^2)}.$$

One can verify that $\theta_H > \theta_L$ by noticing that we can write θ_L as

$$\theta_L = \frac{2\beta_B(1 + \rho\gamma_B + \rho^2 r(\alpha^*))}{(\beta_A^2 + \beta_B^2)(1 + \rho^2 r(\alpha^*))}$$

If $\theta > \theta_H$ then $G(\bar{e}_A, \bar{e}_B) < 0$, meaning that $G(e_A^*, e_B^*) < 0$ for any α. Therefore, we have

$$\frac{d\Pi}{d\alpha} = G(e_A^*, e_B^*)r'(\alpha) \leq 0 \Leftrightarrow \alpha \leq \alpha^* \text{ and } \frac{d\Pi}{d\alpha} = G(e_A^*, e_B^*)r'(\alpha) \geq 0 \Leftrightarrow \alpha \geq \alpha^*.$$

Then the objective function is convex and decreasing in $r(\alpha)$ so that the optimal α is equal to zero.

Solutions for intermediate values of θ

For any $\theta \in]\theta_L, \theta_H[$ the function $G(.)$ is increasing in e_A^* and e_B^* so that there exists a unique $r(\alpha)$ such that $G(e_A^*, e_B^*) = 0$.

It is optimal to select any such $r(\alpha)$ as the first order condition is satisfied at this solution and we have

$$\frac{d^2\Pi}{d\alpha^2} = -(r'(\alpha))^2 \left[\beta_A \frac{\partial G}{\partial e_A^*} + \beta_B \frac{\partial G}{\partial e_B^*}\right] \leq 0.$$

Given that two distinct values of α can lead to the same $r(\alpha)$ and the same effort, the solution in terms of α is not necessarily unique.

Specifically, we have two solutions so long as the lowest value of α solving $G(e_A^*, e_B^*) = 0$ is greater than $\underline{\alpha}$. When it is lower than $\underline{\alpha}$ the solution is unique and converges to 0 as θ increases. ∎

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