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Ambiguity and the Variance of Gambles

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Ambiguity and the Variance of Gambles

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Abstract

Ellsberg's paradox shows that people prefer gambles with known probabilities to those where they are uncertain. Standard explanations rule out risk aversion by appealing to Savage's (1954) subjective expected utility theory but this axiomatic approach leaves open other interpretations of the evidence. We provide a simpler argument: a routine application of the law of total variance shows that the variance of the payoff from a binary gamble is determined entirely by the mean probability belief, not by uncertainty about those beliefs. Ellsberg-type choices are not consistent with rational mean–variance evaluations of risk.

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1. Introduction

Daniel Ellsberg's (1961) thought experiments provided one of the earliest and most enduring challenges to expected utility theory. When asked to pick either red or black from an urn, he found that people preferred betting on a known 50/50 option rather than on an urn with unknown composition, even though expected values were identical. Ellsberg did not conduct formal experiments but his finding has been replicated many times since with proper experimental designs.[1]

Ellsberg's findings are often described as a paradox because they violate the predictions of expected utility theory. In particular, he showed that decision-makers who satisfied the axioms of Savage's (1954) subjective expected utility theory should not have displayed the observed preference. Savage's framework implied that people facing a decision involving an unknown probability should substitute their mean estimate and act as if it were a known probability. In the red–black case, this would mean treating the unknown urn as if it were 50/50, leaving them indifferent between the two options. The fact that people were not indifferent suggested that their behavior could not be explained by risk aversion, but instead reflected an aversion to ambiguity itself.

One way to put this argument is: "That's ambiguity aversion, not risk aversion, because a risk averse person that followed Savage's axioms would be indifferent." But this line of reasoning has serious limitations. At best, it tells us only that people are not behaving in accordance with Savage's axioms. That leaves open many possibilities. Ambiguity aversion is one, but shifting or context-dependent preferences or simple inconsistency are others. And it is hardly persuasive in practice. Most people have never heard of Savage or his axioms, and believe their preference for the known 50/50 bet is simply sensible risk avoidance. Indeed, Ellsberg himself remarked that very few people who picked the option with known probabilities changed their minds after he explained the logic of Savage's axioms to them. He closed his paper by reflecting on those who had preferred the 50/50 bet:

> for their behavior in the situations in question, the Bayesian or Savage approach gives wrong predictions and, by their lights, bad advice. They act in conflict with the axioms deliberately, without apology, because it seems to them the sensible way to behave. Are they clearly mistaken?

Convincing arguments should actually convince people. In practice, telling people "your preferences violate these old axioms!" has not persuaded many that their preferences represent anything other than a perfectly reasonable aversion to risk.

This paper develops a more direct and intuitive way to rule out rational models of risk aversion as an explanation for the choices made in Ellsberg-style experiments. Suppose we describe risk attitudes in the simple mean–variance terms made famous by Markowitz (1952): people like higher

[1] See Camerer and Weber (1992) for a survey.

expected payoffs and dislike higher variance. In our setting, the relevant mean and variance are the subjective moments implied by the agent's probability beliefs. A simple application of the law of total variance shows that the subjective variance of the payoff from a binary gamble depends only on the mean probability assessment, not on how uncertain one is about that probability. In other words, ambiguity does not raise the variance of payoffs. In this precise sense, more ambiguous options are not riskier. The result is a generalization of a property of the commonly-used Beta-Binomial model in Bayesian statistics and a specific example of a well-known property of mixture distributions. To my knowledge, however, this simple but striking property of Bernoulli payoffs has not been highlighted before.

Our result makes Ellsberg's paradox sharper. People may perceive more ambiguous gambles as somehow riskier, but in mean–variance terms they are not riskier at all. The preference for known probabilities cannot be explained by any standard rational form of risk aversion — not even the most intuitive mean–variance version.

The rest of the paper is organized as follows. Section 2 introduces our generalized Ellsberg example and briefly discusses examples from the world of sports betting. Section 3 provides the main result. Section 4 contains some concluding thoughts.

2. Setup and Evidence

People commonly have to take decisions to choose among uncertain options where their confidence in the underlying probabilities varies. In our example, people are offered a \$1 gamble that will return \$$D$ if successful and zero otherwise, consistent with decimal odds of D. The probability of success is P which is unknown with mean p. Consider the choice between multiple options where p is the same but the variance of beliefs about P differs.

A wide range of evidence from Ellsberg-style experiments points in the same direction: people prefer gambles that they perceive to have less uncertainty about the underlying probability. For example, Fox and Tversky (1995) asked Stanford graduate students how much they would pay to play a game that would pay \$100 if the temperature next week was above a specific level in San Francisco and Istanbul, and also to play the same game with the temperature being at or below this level. Participants were willing to pay more to play the San Francisco games, most likely reflecting more knowledge about local weather and a reduced range of values for their subjective probabilities. Halevy (2007) also reported experimental evidence that when faced with two gambles, one of which adds a mean-preserving spread to uncertainty about the probability of success, people prefer the less ambiguous option.

Beyond experiments, sports betting provides obvious examples of varying levels of ambiguity. Bookmakers can accept bets on the Super Bowl, confident in their assessment of the likely outcomes but they are less confident about third–tier Norwegian soccer.

The evidence shows that bookmakers set a higher built-in "hold" or profit margin on bets on more obscure events. For example, Whelan (2025) documents that bookmakers' margins for European soccer leagues are systematically lower for high-profile leagues, with the English Premier League having the lowest margins and the Spanish La Liga having the next lowest. Similarly, for tennis, margins are lower for Grand Slam events and fall as tournaments progress. There may be other explanations than risk or ambiguity aversion for these patterns. For example, it may cost as much to research a Premier League game as it does a game in a more obscure league but the Premier League game will generate much higher volumes. But anecdotal evidence points to bookmakers viewing less well-known events as being riskier.[2]

3. Result

Let Y be an outcome that is either 1 in the case of success or 0 in the case of failure. If the success chance were known to be p, then Y follows a Bernoulli distribution and the familiar formula says

$$\mathrm{Var}(Y \mid P = p) = p(1-p). \tag{1}$$

But in our setting, P is uncertain. You have a subjective distribution of beliefs over possible values of P with mean $\mu = \mathbb{E}[P]$. In this case, you can use the Law of Total Variance to decompose the variance into two parts:

$$\mathrm{Var}(Y) = \underbrace{\mathbb{E}\big[\mathrm{Var}(Y \mid P)\big]}_{\text{average variability } \textit{within} \text{ each } p\text{-world}} + \underbrace{\mathrm{Var}\big(\mathbb{E}[Y \mid P]\big)}_{\text{variability } \textit{across } p\text{-worlds}}. \tag{2}$$

For a binary Y this becomes

$$\mathrm{Var}(Y) = \mathbb{E}\big[P(1-P)\big] + \mathrm{Var}(P). \tag{3}$$

Expanding each term gives

$$\mathbb{E}[P(1-P)] = \mathbb{E}[P] - \mathbb{E}[P^2] = \mu - \mathbb{E}[P^2], \tag{4}$$

$$\mathrm{Var}(P) = \mathbb{E}[P^2] - \mu^2. \tag{5}$$

[2]Marco Blume, trading director of Pinnacle (one of the world's largest bookmakers) did a podcast interview in 2021 explaining his firm's approach to setting odds. He described their strategy of setting higher margins for more obscure events. The interview can be found at https://www.youtube.com/watch?v=8VpaC-4qYQY and https://www.youtube.com/watch?v=U529G$_{P}RrR$0

Adding them, the two $\mathbb{E}[P^2]$ terms cancel.

$$\mathrm{Var}(Y) = (\mu - \mathbb{E}[P^2]) + (\mathbb{E}[P^2] - \mu^2) = \mu(1-\mu). \tag{6}$$

The dependence on the dispersion in beliefs about P drops out completely. Only the mean belief μ matters.

In our example from the previous section where the payoff for a gamble is $X = DY$, then the variance of the payout is

$$\mathrm{Var}(X) = D^2\,\mu(1-\mu). \tag{7}$$

so the result generalizes to gambles with non-unit winning payoffs.

To my knowledge, this particular result on the variance of Bernoulli gambles has not featured in the economics or decision theory literature before, but it is related to two well-known results.

First, it is a generalization of a well-known property of the Beta–Binomial predictive distribution in Bayesian statistics, based on matching a prior $P \sim \mathrm{Beta}(\alpha, \beta)$ for the probability of success with a Binomial distribution. Bernardo and Smith (1994, p. 117) give the general variance formula for the Beta–Binomial predictive distribution with n trials. When $n = 1$, the prior mean is $\mu = \alpha/(\alpha+\beta)$ and the predictive variance is $\mathrm{Var}(Y) = \mu(1-\mu)$. Our result shows that this same property holds not just for Beta priors but for any distribution of beliefs about P.

Second, our result is a special case of the general variance formula for mixture distributions. If a random variable Y is drawn from distribution F_i with probability w_i, then the variance of the mixture is

$$\mathrm{Var}(Y) \;=\; \sum_i w_i\big(\sigma_i^2 + \mu_i^2\big) - \mu^2 \tag{8}$$

where μ_i and σ_i^2 are the mean and variance of F_i, and $\mu = \sum_i w_i \mu_i$ is the overall mean.[3] In our case, using $\sigma_i^2 = \mu_i\,(1-\mu_i)$ gives our result that $\mathrm{Var}(Y) = \mu(1-\mu)$.

To illustrate what drives the result, consider two cases. In the first, you are sure the success chance is exactly 0.5, so the total variance is 0.25. In the second, you believe the chance of success is either 0.2 or 0.8, each with probability 0.5. The second case is more ambiguous but the payoff variance is the same in both cases. When $P = 0.2$ the variance of the Bernoulli outcome is $(0.2)\,(0.8) = 0.16$, which is lower than when $P = 0.5$. The same is true when $P = 0.8$, where the variance is also 0.16. Averaging these lower within-world variances with the variability across worlds produces a cancellation: the total variance is again 0.25, exactly the same as in the known $p = 0.5$ case.

While mixing of continuous distributions typically produces new distributions with complex shapes, a mixture of Bernoulli distributions still produces an outcome that is either one or zero. This

[3] See Fabio Corona's lecture notes on mixture distributions available on his Github site and the Wikipedia entry on "Mixture distribution" for derivations of this formula.

means the outcomes are still characterized by a Bernoulli distribution and with an expected value of μ, its variance must be $\mu(1-\mu)$. Indeed, since the Bernoulli distribution has only one parameter, all higher moments of the mixture distribution are the same as the distribution with a known success rate.

4. Discussion

The result we have discussed shows that Ellsberg-style findings are inconsistent not only with Savage's axioms but also with rational mean–variance evaluations of risk. The result provides a simpler way of illustrating why rational people should not be averse to gambles with ambiguous probabilities. Ambiguity does not increase payoff variance and rational evaluations should not treat it as if it did.

I conclude with some speculation for further research. Anecdotally, it has been my experience that the variance result presented here tends to surprise people. They expect higher uncertainty about the probability of success to translate into higher variance for the payoff. Their intuition is not unreasonable: for most continuous distributions, uncertainty about a parameter does raise variance. For example, consider a Normal distribution with mean 0.5 and variance of 0.1. If we change to a situation where the variance is the same but the mean is equally likely to be 0.2 or 0.8, the variance rises to 0.19. This is perhaps the intuition people transfer to Bernoulli gambles, but in that setting it is mistaken.

That the variance result surprises people could point to another interpretation of Ellsberg-style results, perhaps more subtle than simply an aversion to ambiguity itself. Human intuition may not be well equipped to compute variances under ambiguous probabilities. Part of what is labeled "ambiguity aversion" may simply be that people perceive ambiguous gambles as riskier, because they do not apply the law of total variance correctly. Ellsberg experiments may therefore capture both genuine aversion to ambiguity and a more basic cognitive difficulty in evaluating risk under ambiguity.

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