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Regulatory Compliance in the Automobile Industry

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Regulatory Compliance in the Automobile Industry

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Abstract: This paper analyzes compliance with emission regulations, focusing on investment in innovation and cheating as primary strategies. Firms prioritize developing a truly compliant technology but may activate a cheating device depending on the outcome of innovation, the monitoring system in place, and the size of compliance costs. Successful innovation achieves compliance at lower costs, while undetected cheating creates the appearance of compliance while eliminating all compliance costs. Relying on the automobile sector as a case study, we explore the path-dependent nature of cheating and investment decisions and demonstrate that investment in innovation and cheating are strategic substitutes. Firms invest less in innovation when they anticipate using a cheating device, either systematically or as a fallback when innovation fails. We derive policy recommendations from comparative statics based on compliance costs, enforcement efforts, and competition. Finally, we assess whether the increased ease of cheating benefits the automobile industry and show that firms may continue to rely on this strategy even when it is inefficient.

Keywords: Environmental regulation, compliance, automobiles, fuel economy, innovation, cheating.

JEL codes : L5, Q5, O3, D21

1. Introduction

Across many sectors, in different regions of the world, firms face increasingly stringent environmental regulations in terms of compliance, emissions limits and technological requirements, aiming to address worsening environmental issues (Kozluk and Garsous, 2016). For instance, the European Union's Circular Economy Action Plan sets strict recycling and waste reduction targets for manufacturing industries.[1] Similarly, the European Farm to Fork Strategy aims to reduce pesticide use by 50% by 2030.[2] In the automobile industry, air pollutant emissions from passenger cars have been regulated in the United States (U.S.) since 1963 and in Europe since 1977, with regulations tightening over time. For example, NOx emissions standards for new diesel cars reduced significantly between 2000-2014, from 0.5g/km to 0.08 g/km.[3] Emissions standards for nitrogen oxides (NOx), hydrocarbons, and carbon monoxide (CO) apply to both petrol and diesel vehicles, with additional particulate limits for diesel vehicles.

The most common and desirable compliance strategy consists in investing in innovation to try and reach the new standards at a the lowest possible cost. However, as regulations become increasingly stringent, firms have a greater incentive to explore alternatives including deceptive practices. For instance, some marine engine manufacturers have been accused of using software or hardware solutions to enable ships and boats to emit more pollutants than allowed during regular operations.[4] Similarly, the energy performance of a significant share of European home appliances was found to be inconsistent with their label (Baton et al., 2017). Reynaert and Sallee (2021) provide evidence that gaming or cheating, understood as manipulating emissions signals, has risen significantly in the

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[1] See https://environment.ec.europa.eu/topics/circular-economy_en

[2] https://food.ec.europa.eu/plants/pesticides/sustainable-use-pesticides/farm-fork-targets-progress_en#:~:text=The%20Farm%20to%20Fork%20and,more%20hazardous%20pesticides%20by%202030

[3] European Environment Agency (2016) *Comparison of NOx emission standards for different Euro classes* Available: https://www.eea.europa.eu/en/analysis/maps-and-charts/comparison-of-nox-emission-standards

[4] See https://www.hydrotech-group.com/blog/thousands-of-ships-fitted-with-cheat-water-treatment-devices-cause-ocean-pollution-and-violate-emissions-regulations.

automobile industry over the past 15 years. In September 2015 the U.S. Environmental Protection Agency issued a Notice of Violation to the Volkswagen Group, stating that approximately 480,000 Volkswagen and Audi diesel automobiles were equipped with emissions-compliance defeat devices. Since then, similar allegations have surfaced against other automobile manufacturers (The ICCT, 2017; Lyon, 2018; Fleetnews 2021, US Department of Justice, 2022). The Dieselgate scandal has shed light on the strategic decisions regarding the installation and activation of cheating devices, as well as their impact on investments in innovative abatement technologies.

In this paper we propose a positive analysis focusing on investment in innovation and cheating as the primary compliance strategies. Our approach incorporates factors salient to the automobile industry but it is defined broadly enough to apply to circumstances in other industries. This paper distinguishes itself from the literature by considering the path-dependent nature of cheating and investment decisions. Specifically, once new emission standards are imposed, firms decide on their level of investment in innovation. Then, after assessing whether an innovative solution to achieving compliance at a lower cost is feasible, the firms decide whether to activate a cheating device. In line with some evidence, we consider that cheating devices are inherently available to the firms.[5] The sequential model we propose enables us to capture the strategic interaction between cheating and innovation, with a particular focus on investment in innovation in a context where firms perfectly anticipate the possibility of relying on a cheating device.

The efficiency and effectiveness of environmental policies in reducing pollution and encouraging compliance have been widely studied over the past 50 years (e.g., Baumol and Oates, 1977; Newell and Stavins, 2003; Requate, 2005; Goulder and Parry, 2008; Sterner and

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[5] In recent years, in the automobile industry, cheating has become easier to execute and more difficult to detect due to the widespread use of computerized engine systems, where most functions are controlled by software (Andersen et al, 2018). The defeat device used by the Volkswagen Group was embedded in the engine's control software and was programmed to disable emissions controls during real-world driving.

Robinson, 2018). While economists highlight the cost-efficiency of market-based instruments, policymakers have favoured command-and-control (CAC) regulations (Requate, 2005; Vollebergh and van der Werf, 2014). Thus, some studies examine compliance and enforcement through fines and monitoring (Malik,1992, and Heyes, 2000). Others explore pollution taxes and permit-based regulations (Macho-Stadler and Pérez-Castrillo, 2006, Macho-Stadler, 2008, and Coria and Villegas-Palacio, 2010 and 2014). However, none of these studies consider the possibility of cheating as a compliance strategy.

The impact of environmental policies on innovation and eco-innovation has received considerable attention in the literature. Porter and van der Linde (1995) postulate that the question of the economic burden placed on industry by environmental regulations has been incorrectly framed. In a dynamic world, where firms adapt and innovate, they and others argue that well-designed environmental standards can trigger innovation and enhance firms' competitiveness (Ashford et al., 1985, Dechezleprêtre and Sato, 2017; Popp, 2019; Jaffe et al., 2002; Ramanthan et al., 2017). In the automobile sector, significant evidence shows that advances in combustion engine development and emissions control technology have been driven by increasingly stringent environmental standards (Knecht, 2008; Vollebergh and van der Werf, 2014). Different policy instruments can trigger different types of innovation as documented in Bergek and Berggren (2014) who show that general regulatory instruments, such as those in the automobile sector, encourage modular innovation that involve "*additions or substantial changes to the core design concept of one or more component*". Interestingly Yao (1988) and Puller (2006) show that investment in innovation can also influence standard settings. None of these studies consider cheating as a compliance strategy.

Our approach is closer to Malik (1990), Reynaert and Sallee (2021) and Reynaert (2021) in that we consider cheating as a potential compliance strategy. Malik (1990) considers a setting where the decision to cheat is not strategic: firms are de facto offenders who differ in what they stand to gain when not caught cheating. Reynaert and Sallee (2021) examine car manufacturers that can deceive consumers by misreporting certain vehicle characteristics. The harm caused to duped consumers is then contrasted with the cost savings from regulatory avoidance. Finally, Reynaert (2021) considers a representative firm that can employ multiple compliance strategies. Among these, cheating and technology adoption allow the firm to advertise lower fuel consumption. Using data on European emission standards from 1998 to 2011, the author demonstrates that these two strategies are used more prominently than mix-shifting and downsizing, leading to some welfare losses.

In our setting two competing firms subject to new emission standards. In line with Yao (1988), we assume that compliance is achievable but requires incurring fixed and marginal compliance costs, which we separate into unavoidable costs and those that can be reduced through innovation. A failure to innovate can increase fixed costs due to the need for more drastic measures to meet the standards as well as marginal costs due to the requirement to install more expensive abatement technologies. The success of innovation is uncertain and positively correlated with the level of investment undertaken and a firm's capability to innovate. Once firms know whether innovation was successful, they can activate a cheating device to make their vehicles appear compliant.[6] Thus, we consider that firms prioritize the option of developing a truly compliant technology. Cheating is considered a post-innovation strategy that creates the appearance of compliance without incurring any compliance costs, as long as the device remains undetected. If the device is detected, the firm faces a penalty and must cover compliance costs, which depend on whether it had successfully innovated. Regulators enforce standards through monitoring, allowing them to detect the device with some probability and penalize cheating. Finally, firms compete in prices, offering products that are, or appear to be, compliant. These products are otherwise horizontally differentiated based on their appearance and other features (Hotelling, 1929).

As can be expected, we find that firms are more likely to activate the defeat device when they fail to innovate. However, the decision to activate the device (post-innovation) ultimately depends on the size of the regulator's enforcement efforts (defined as the expected penalty fee which accounts for the risk of being caught) *relative to* the compliance costs.

[6] In the case of the Volkswagen, this sequence of events is in line which what is depicted Ewing (2017): the company decided to leave the device in place even after it had successfully developed a technology called BlueMotion.

Thus, we provide evidence that the increased reliance on cheating devices documented in the literature can result from a combination of high compliance costs and the fact that cheating has become more difficult to detect. This supports findings in Hu et al. (2021) arguing that standards that are difficult to reach should come with increased monitoring/penalty.

We demonstrate that firms invest more in innovation when facing higher compliance costs (both fixed and marginal) that are avoidable with successful innovation. However, comparative statics on optimal investments reveal that other parameters, including the regulator's enforcement efforts, have a non-monotonic impact. For instance, a higher fee or a greater ability to detect devices can have a detrimental impact on the investment in innovation. This happens in a context where firms use a mixed strategy activating the cheating device. In this case, higher enforcement efforts generate higher profits when both firms fail to innovate because it lowers the weight assigned to cheating. Having less to lose when innovation fails triggers lower investments. Finally, we also show that firms invest less in innovation when expect that they will activate a cheating device. Said differently: cheating and investing in innovation are shown to be strategic substitutes.

While we show that competition can reduce the need for monitoring to induce true compliance, we also establish that it has a non-obvious impact on the investment strategy. As products become closer substitutes, firms with a high capability to innovate lower their investments in innovation while weak innovators increase their investment. Finally, we evaluate whether the increased ease of cheating benefits the automobile industry. We consider this in the context of firms having the option to commit to honesty, such as through voluntary agreements. Using simulations, we show that not committing always emerges as a dominant strategy. The equilibrium is efficient (it maximizes the firms' profits) when the enforcement efforts are low. However, as the enforcement efforts increase the equilibrium forms a prisoner's dilemma implying that firms tend to hold on to the possibility to cheat even when it is inefficient.

The next section presents the model and discusses some of the assumptions. Section 3 discusses the motivation for the underlying set-up. Section 4 characterizes the outcome of price competition; section 5 examines the decision to cheat and Section 6 is devoted to the investment decision . Section 7 focuses on the policy implications and Section 8 concludes.

2. The Model

We consider a game initiated by a regulatory agency that sets emission standards for two car manufacturers (Firm 1 and Firm 2). While automobile manufacturers typically offer an entire fleet of heterogenous vehicles, we simplify the analysis by assuming that Firm i supplies product i $(i = 1,2)$. Consumers perceive products 1 and 2 as horizontally differentiated, which is captured using a Hotelling (1929) model with Firms 1 and 2 located at the extremities of a [0,1] line representing consumers' preferences. There is a mass of one of consumers, whose preferences are uniformly distributed over [0,1]. Given prices p_1 and p_2, a consumer located at $x \in [0,1]$ purchases a car from Firm 1 if and only if

$$v - p_1 - tx \geq v - p_2 - t(1-x) \text{ and } v - p_1 - tx \geq 0.$$

Similarly, the consumer purchases a car from Firm 2 if and only if:

$$v - p_1 - tx < v - p_2 - t(1-x) \text{ and } v - p_2 - t(1-x) \geq 0.$$

The parameter $v > 0$ denotes the value a consumer places on obtaining a car that matches his ideal specifications, while $t > 0$ captures the degree of differentiation (and thus, the intensity of competition). We assume that v is large enough so that, in equilibrium, we always have $v - p_1 - tx > 0$ and $v - p_2 - t(1-x) > 0$ and so that firms generate sufficient revenues to cover their overall compliance costs. Under these assumptions, the demand function for Firm i's product is given by:

$$D_i(p_i, p_j) = \frac{1}{2t}\left(t + p_j - p_i\right), \quad i, j = 1,2, i \neq j. \tag{1}$$

To simplify the analysis and without loss of generality, we assume that the firms' production technology is such that total manufacturing costs are linear, meaning that the average and marginal costs are equal. Specifically, we assume that these production costs are zero at the time the new emission standards are announced.

As is often the case, new regulations come into force a few years after being announced. During this period, firms must decide on their strategies to achieve compliance. In line with Yao (1988), we assume that the new standards are achievable, albeit with associated compliance costs. The level of these costs depends on the strategy a firm adopts to achieve compliance and whether the strategy is successful.

The most immediate decision for each firm is the amount devoted to research and development (R&D) activities. The success of R&D activities is uncertain. Let $I_i \geq 0$ denote firm i's investment in R&D. The probability of success is captured by a function $P(\theta_i, I_i)$ where θ_i denotes firm i's capability to innovate, $i \in \{1,2\}$. This exogenous parameter reflects a firm's idiosyncratic characteristics that influence the probability of success. Such characteristics can include the quality of its R&D team, its level of absorptive capacity, its innovation record, and perhaps the type of vehicles it aims to sell.[7] We make the following assumptions regarding the function $P(\theta, I)$:

(i) For any $(\theta, I), P(\theta, I) \in [0,1]$ with $P(0, I) = 0$,

(ii) $P(.)$ is increasing and concave with respect to θ,

(iii) $P(.)$ is increasing and concave with respect to I,

(iv) $P_I > 0$ for all $\theta > 0$ and such that $\lim_{I \to 0} P_I = +\infty$ and $\lim_{I \to +\infty} P_I = 0$.

(v) $P_{\theta I} > 0$ for all $\theta > 0$

Assumption (i) reflects the fact that the function $P(\theta, I)$ is a probability and that innovation success requires some level of innovative capability. Assumption (ii) captures the convention that a higher type (θ) indicates greater innovative capability for any given investment level. Assumptions (iii) and (iv) ensure that the optimization problem is concave in I, as the marginal increase in the probability of success decreases with higher investment. Assumption (v) captures the fact that the increase in the probability of success due to a marginal increment in investment is greater for firms with higher innovative capability.

[7] The concept of absorptive capacities refers to a firm's ability to identify, assimilate, transform, and apply external knowledge, research, and practice.

We assume that the outcome of innovation is perfectly known to all parties. If a firm innovates successfully, we consider that it only incurs a non-negative fixed compliance cost $c \geq 0$. If a firm fails to innovate, its marginal compliance cost increases to $\rho \geq 0$, while its fixed compliance costs increase to $(c + d)$, where $d \geq 0$. To ensure that we have a positive demand for each firm's output in equilibrium, we impose $\rho < 3t$.[8]

Once the innovation outcome is known, the firms face a final "compliance" decision: whether to activate a defeat device. Cheating provides an opportunity to avoid all compliance costs, including c, provided that the device is not found.[9] We assume that installing such a device is costless. Firms' actions at this stage are labelled "*ON*" and "*OFF*", where "*ON*" refers to activating the defeat device. Naturally, this strategy is not disclosed publicly. When the device is activated, the vehicle appears compliant during regulatory testing. Consumers cannot verify this claim and, even if skeptical, lack the technology to detect deception.

Finally, before the new automobiles are introduced to the market, the regulatory agency inspects the final products. Let $\gamma \in]0,1]$ capture the effectiveness of the defeat device. The parameter γ is exogenous, and a higher γ indicates a greater difficulty for regulators to detect the device.[10] To simplify the analysis, we assume with a probability $(1-\gamma)$ all activated cheating devices are detected.[11] If a firm is caught cheating, it must disable the device and take actions to achieve true compliance. In this case, compliance costs depend on whether the firm successfully innovates, as described earlier. Additionally, the firm faces a fixed penalty $\Gamma \geq 0$, which can include reputational damage and monetary fines.

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[8] We assume that the compliance costs, c, d and ρ, are identical for both firms, which is a simplifying assumption that allows us to solve the model. While it could be interesting to allow for idiosyncratic compliance costs in a setting where firms are either perfectly or imperfectly informed about their rival's costs, this issue is beyond the scope of our current analysis.

[9] If a firm decides to cheat post-innovation, and if innovation was successful, it can still benefit from its discovery avoiding costs d and ρ. If we consider $c = 0$ we face a situation where cheating and investing in innovation are more aligned as both strategies enable the firms to eliminate additional compliance costs that occur when innovation fails.

[10] A higher γ also implies weaker regulatory monitoring as detection becomes less likely.

[11] If the regulatory agency finds the device in the vehicles produced by Firm 1, it automatically inspects all the other vehicles on the market and is able to find the defeat device in the vehicles produced by Firm 2 if they are activated.

We summarize the definitions of all the parameters in Appendix A. Table 1 summarizes the compliance costs, net of the endogenous investment in innovation I_i, based on the decision to activate the device and on the outcomes of innovation and of monitoring.

	INNOVATION OUTCOME			
	SUCCESS		**FAILURE**	
DEVICE:	**ON**	**OFF** Marginal cost: 0 Fixed cost: c	**ON**	**OFF** Marginal cost: ρ Fixed costs: $c + d$
CAUGHT:	**YES** Marginal cost: 0 Fixed costs: $c + \Gamma$	**NO** Marginal cost: 0 Fixed costs: 0	**YES** Marginal cost: ρ Fixed costs: $c + d + \Gamma$	**NO** Marginal cost: 0 Fixed cost: 0

Table 1: Firm i's marginal and fixed costs (net of the investment in innovation) under all possible strategies and outcomes.

The timing of the game is as follows:

T=0 The regulatory agency announces the new emission standards.

T=1 Firms simultaneously and non-cooperatively decide how much to invest in innovation.

T=2 Innovation investments are sunk, and the success or failure of the innovation becomes public knowledge.

T=3 Based on this information, the firms decide whether to activate a defeat device simultaneously and non-cooperatively. The activated defeat device is detected with a probability of $(1 - \gamma)$.

T=4 Firms compete in prices and their profits realize.

3. Discussion of the model setup

The model setup is constructed with the automobile industry in mind where regulators impose emissions standards to mitigate the externalities associated with air pollution and greenhouse gas emissions. Consumers assume that vehicles are compliant with emissions standards and their purchasing decisions are based on prices as well as characteristics such as fuel economy, appearance and other aesthetical preferences, all captured by t.

With stringent emissions standards, firms often need to invest in innovation to search for and develop an abatement technology. This investment is captured by the variable I in the model. In the case of NOx emissions reduction in diesel vehicles, this may involve investing in the development of new technologies such as Exhaust Gas Recirculation (EGR), Select Catalytic Reduction (SCR), or a Lean NOx Trap (LNT) (Andersen et al, 2018). However, there is generally a trade-off between NOx reduction and fuel economy, as most emission control technologies come with an increase in fuel consumption by approximately 18-21% for diesel and hybrid diesel engines (Andersen, et al., 2018). Bresnahan and Yao (1985) show that there is also a trade-off between emissions reduction and drivability. Therefore, a successful innovation is one that develops an emissions reduction technology without compromising on fuel economy and/or drivability. With this in mind, the parameter c may include costs of establishing a new engineering division for technology development or setting up a new assembly line for production. The additional fixed cost d can represent additional costs required if the investment in innovation is unsuccessful, either because the technology fails to sufficiently reduce NOx emissions to meet the standards or because it meets the standard but introduces excessive compromises. The fixed cost d could also reflect the need to rely on strategies that firms would have avoided had innovation succeeded, such as downsizing or mix-shifting. When innovation fails, we consider that the marginal cost of production (ρ) may also increase. This can occur if additional pieces of equipment are required in each vehicle to ensure compliance. For example, in the Volkswagen case, the firm's innovation was insufficient to comply with the NOx emissions standards. As a result, the company initially purchased a control technology from Mercedes-Benz, the BlueTec system, and installed it in each diesel vehicle across particular models. However, they later rejected BlueTec due to its inconvenience and high cost, opting instead to install a lean-NOx-trap (LNT), which reduced fuel efficiency during operation.[12] In this paper, we do not address the optimal stringency of the emissions standards. Nevertheless, higher compliance costs (c, d and ρ) could also be interpreted as reflecting more stringent emissions policies.

We assume that the defeat device is easy to activate and costless. Modern vehicle engines are equipped with electronic devices designed to optimize the combustion process. These can be easily programmed to operate the engine differently in real-world conditions compared to during compliance testing. The Dieselgate scandal shows that Volkswagen opted for a cheaper NOx control technology rather than the more expensive BlueTec system. This cheaper technology appeared to comply with emissions standards only during test conditions. On the road, the defeat device deactivated at least part of the NOx emissions controls to enhance fuel economy and vehicle performance. When the discrepancy between test and on-road emissions was brought to light, Volkswagen was forced to deactivate the device and faced over $30 billion in fines in the US.

Finally, regarding enforcement, we assume that cheating devices are detected with a probability of $(1-\gamma)$. This assumption implicitly captures the possibility that the discovery of a cheating device in one firm's vehicles will prompt the regulator to scrutinize the automobiles produced by rival firms more closely. In real life, on-road emissions controls for both new and older vehicles have become more widespread since 2015. Such testing is now mandatory in the EU and some US states since the Dieselgate scandal.[13] In our model, we consider that the regulator can test vehicles before they are offered to consumers. We could instead assume that vehicles are tested at some random point after they are sold, or on several occasions. This would complicate the expression for profits while adding little to the

[12] Ewing, J. (2015) 'VW Says Emissions Cheating Was Not a One-Time Error', *New York Times*, 10th December 2015. https://www.nytimes.com/2015/12/11/business/international/vw-emissions-scandal.html

[13] Regulation 2018/858 of the European Parliament and of the Council of 30 May 2018 on the approval and market surveillance of motor vehicles and their trailers, and of systems, components and separate technical units intended for such vehicles, amending Regulations (EV No 715/2007 and EV No 595/2009 and repealing Directive 2007/46/EC.

fact that firms who cheat take the risk of facing some penalties once their product is on the market.

4. Market competition and equilibrium prices

We solve for a subgame perfect equilibrium, where firms perfectly anticipate their future decisions. Therefore, we start with the last stage. According to the model setup in Section 2, the demand function for firm i's product, defined in Equation (1), is:

$$D_i(p_i, p_j) = \frac{1}{2t}(t + p_j - p_i).$$

We solve for the Bertrand-Nash equilibrium prices. Given any price set by its competitor, firm i chooses p_i to maximize $\pi_i = (p_i - \sigma_i)D_i(p_i, p_j)$. It is straightforward to show that the equilibrium prices and profits (net of any fixed compliance costs and potential penalties) are such that

$$p_i^* = t + \frac{1}{3}(2\sigma_i + \sigma_j), \tag{2}$$

and

$$\pi_i^* = \frac{1}{2t}\left(t + \frac{1}{3}(\sigma_j - \sigma_i)\right)^2, i, j = 1,2 \text{ and } j \neq i, \tag{3}$$

where $\sigma_i = 0$ if firm i innovates successfully or if it activates the device and is not caught, and $\sigma_i = \rho$ otherwise. If the firms have the same marginal cost, each firm earns profits (net of any fixed compliance costs) equal to $\pi^* = \frac{1}{2}t$. If firm i has a cost advantage such that $\sigma_i = 0$ and $\sigma_j = \rho$, the equilibrium profits are given by:

$$\pi_i^* = \frac{1}{2t}\left(t + \frac{1}{3}\rho\right)^2 \text{ and } \pi_j^* = \frac{1}{2t}\left(t - \frac{1}{3}\rho\right)^2 \quad i, j = 1,2 \text{ and } j \neq i.$$

These equilibrium profits can also be rewritten as:

$$\pi_i^* = \pi^* + \Delta, \text{and } \pi_j^* = \pi^* - \nabla,$$

where:

$$\Delta = \frac{\rho}{18t}(6t + \rho) \text{ and } \nabla = \frac{\rho}{18t}(6t - \rho).$$

The discrepancy in profits arises only if the innovation leads to a lower marginal cost (i.e. if $\rho > 0$). If the innovation reduces only the fixed compliance costs, then the net profits are always equal to π^*, and the firms may differ only in terms of the fixed costs they incur. As the rest of the paper will show, if $\rho = 0$, all compliance decisions are strategically independent: each firm has a dominant strategy, and its course of actions does not depend on what its rival does. Some interdependence occurs if $\rho > 0$. In this case, the compliance decisions become strategically interlinked.

In the absence of any asymmetry of information, one could assume that the regulator can use the realized profits to detect cheating. Here we consider that a firm can only be penalized provided the defeat device is detected. Effectively, the profits that are observed by the regulator could include the realization of a random variable with zero mean. Thus, and for instance, the profits gathered by Firms 1 and 2 when do not cheat and when only Firm i innovates successfully are realizations of $(\pi^* + \Delta - c + \tilde{\varepsilon}_i)$ and $\left(\pi^* - \nabla - c - d + \tilde{\varepsilon}_j\right)$ where $\tilde{\varepsilon}_i$ and $\tilde{\varepsilon}_j$ capture random shocks with zero mean.

5. Decision to activate a defeat device

At this stage, each firm knows whether innovation has been successful for itself and for its competitor. Therefore, the firms are aware that they are in one of four possible states of the world. Let S denote success and F failure. The vector $O = (O_1, O_2)$ represents the innovation outcomes for firms 1 and 2, respectively, where $O_i \in \{S, F\}$. Specifically, the four possible states are (S, S), (S, F), (F, S), and (F, F). In this context, the state (S, F) refers to the situation where only firm 1 has successfully innovated. In each of these states, firms decide whether to activate the defeat device. We assume that a firm will not activate the device if it is indifferent between doing so and not doing so.

Lemma 1: *In the states (S, S), (S, F), and (F, S), the two firms have a dominant strategy which depends on the value of $(1 - \gamma)\Gamma$ and on whether innovation is successful. Upon successful innovation, "ON" is a dominant strategy if $(1 - \gamma)\Gamma < \gamma c$, and "OFF" is a dominant strategy if*

$(1-\gamma)\Gamma \geq \gamma c$. When innovation fails, "ON" is a dominant strategy if $(1-\gamma)\,\Gamma < \gamma(c+d+\nabla)$, and "OFF" is a dominant strategy if $(1-\gamma)\,\Gamma \geq \gamma(c+d+\nabla)$.

In the state (F,F), "ON" is a dominant strategy if $(1-\gamma)\Gamma < \gamma(c+d+\nabla)$, and "OFF" is a dominant strategy if $(1-\gamma)\Gamma \geq \gamma(c+d+\Delta)$. If the parameter ρ is positive so that $\Delta > \nabla$ and if $(1-\gamma)\Gamma \in [\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$ the best reply to "OFF" is "ON", and the best reply to "ON" is "OFF".

Proof: See Appendix B.

Using Lemma 1, Proposition 1 characterizes all equilibria which are all in dominant strategies unless both firms fail to innovate and $\gamma(c+d+\nabla) \leq (1-\gamma)\Gamma < \gamma(c+d+\Delta)$. This condition can only occur if $\rho \neq 0$. It is useful to refer to $(1-\gamma)\Gamma$ as a measure of the regulator's enforcement efforts as it captures the two regulatory tools: the fine and the probability of detecting the device.

Proposition 1: *The decision to activate the device depends on the outcome of innovation as well as on the exogenous parameters, particularly the level of regulator's enforcement efforts captured by $(1-\gamma)\Gamma$. Specifically, the equilibrium strategies are as follows.*

- *If $(1-\gamma)\Gamma < \gamma c$, both firms activate the device regardless of the innovation outcome.*
- *If $\gamma c \leq (1-\gamma)\Gamma < \gamma(c+d+\nabla)$, the firm that fails to innovate (if any) activates the device, while the firm that successfully innovates does not.*

If $\gamma(c+d+\nabla) \leq (1-\gamma)\Gamma < \gamma(c+d+\Delta)$, neither firm activates the device unless both fail to innovate. In the state (F,F), there is a unique mixed strategy equilibrium, where each firm chooses "OFF" with probability $q^* \in [0,1]$ defined as:

$$q^* = \frac{(1-\gamma)\,\Gamma - \gamma(c+d+\nabla)}{\gamma(\Delta-\nabla)}. \tag{4}$$

- *Finally, if $(1-\gamma)\Gamma \geq \gamma(c+d+\Delta)$, neither firm activates the device.*

Proof: See Appendix B.

Table 2 provides a visual representation of the different equilibria based on the innovation outcomes and the level of regulator's enforcement efforts $(1-\gamma)\,\Gamma$. It shows that compliance costs and enforcement efforts do not matter in absolute terms but relative to each other. Firms are more likely to cheat when enforcement efforts are low relative to the compliance costs. That said, as $\gamma \to 0$, meaning the regulatory agency can always detect the device, firms never activate it, regardless of the innovation outcome, even if the penalty is low. Table 2 also highlights the fact that firms are more likely to cheat when innovation fails.

$(1-\gamma)\Gamma \to$	0	γc	$\gamma(c+d+\nabla)$	$\gamma(c+d+\Delta)$
(S,S)	ON	OFF	OFF	OFF
$(S,F)\&(F,S)$	ON	Innovation fails → ON Innovation succeeds →OFF	OFF	OFF
(F,F)	ON	ON	Mixed strategy	OFF

Table 2: Summary of the decision to activate or deactivate the device post-innovation based on the outcome of the innovation and on the value for $(1-\gamma)\,\Gamma$.

The investment decision taken by each firm is based on their ability to perfectly anticipate their decision to cheat. Thus, to evaluate the investment decisions, we must assess and analyze the firm's profits in equilibrium.

When both firms innovate successfully, or when both fail to innovate, their profits at the equilibrium of the final stage are identical, i.e., $\pi_i = \pi_j$. The following table summarizes the profit functions of both firms in the states of the world (S,S) and (F,F), where q^* is given by (4).

$(1-\gamma)\Gamma \downarrow$	$\pi_i(S,S)\ (i=1{,}2)$	$\pi_i(F,F)\ (i=1{,}2)$
$[0,\gamma c[$	$\pi^* - (1-\gamma)(c+\Gamma)$	$\pi^* - (1-\gamma)(c+d+\Gamma)$
$[\gamma c, \gamma(c+d+\nabla)[$	$\pi^* - c$	$\pi^* - (1-\gamma)(c+d+\Gamma)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$\pi^* - c$	$\pi^* - (c+d) - (1-q^*)\gamma\nabla$
$[\gamma(c+d+\Delta), +\infty[$	$\pi^* - c$	$\pi^* - (c+d)$

Table 3: Both firms' profits in the states of the world (S,S) and (F,F).

Both, $\pi_i(S,S)$ and $\pi_i(F,F)$ are continuous and non-increasing as $(1-\gamma)\Gamma$ increases. One can easily verify that since q^* is continuous and such that:

$$(1-q^*) = 1 \text{ at } (1-\gamma)\Gamma = \gamma(c+d+\nabla),$$

$$(1-q^*) = 0 \text{ at } (1-\gamma)\Gamma = \gamma(c+d+\Delta).$$

When firm i successfully innovates while its rival, firm j, fails to innovate, their profits are no longer identical. The middle column of Table 4 shows the profits for the successful firm, while the last column shows the profits for the firm that fails to innovate.

$(1-\gamma)\Gamma \downarrow$	$\pi_i(S,F) = \pi_j(F,S)$	$\pi_j(S,F) = \pi_i(F,S)$
$[0, \gamma c[$	$\pi^* + (1-\gamma)(\Delta - c - \Gamma)$	$\pi^* - (1-\gamma)(\nabla + c + d + \Gamma)$
$[\gamma c, \gamma(c+d+\nabla)[$	$\pi^* + (1-\gamma)\Delta - c$	$\pi^* - (1-\gamma)(\nabla + c + d + \Gamma)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$\pi^* + \Delta - c$	$\pi^* - \nabla - c - d$
$[\gamma(c+d+\Delta), +\infty[$	$\pi^* + \Delta - c$	$\pi^* - \nabla - c - d$

Table 4: Both firms' profits in the states of the world (S,F) and (F,S).

Clearly, $\pi_j(S,F)$ is continuous at $(1-\gamma)\Gamma = \gamma(c+d+\nabla)$ and non-increasing as $(1-\gamma)\Gamma$ increases. By opposition, the function $\pi_i(S,F)$ is discontinuous at $(1-\gamma)\Gamma = \gamma(c+d+\nabla)$ because the rival firm changes its cheating strategy past this point:

- If $(1-\gamma)\Gamma < \gamma(c+d+\nabla)$, the rival firm activates the device when its innovation fails. In this case, firm i can only benefit from a cost advantage when the device is detected.
- If $(1-\gamma)\Gamma \geq \gamma(c+d+\nabla)$, the rival firm never activates the device. Thus, firm i gains additional profits Δ as the only successful innovator.

As illustrated in Figure 1 below, at $(1-\gamma)\Gamma = \gamma(c+d+\nabla)$, the profit function $\pi_i(S,F)$ increases discontinuously by a value of $\gamma\Delta$. There is no discontinuity if $\rho = 0$ because, in this case, $\Delta = \nabla = 0$.

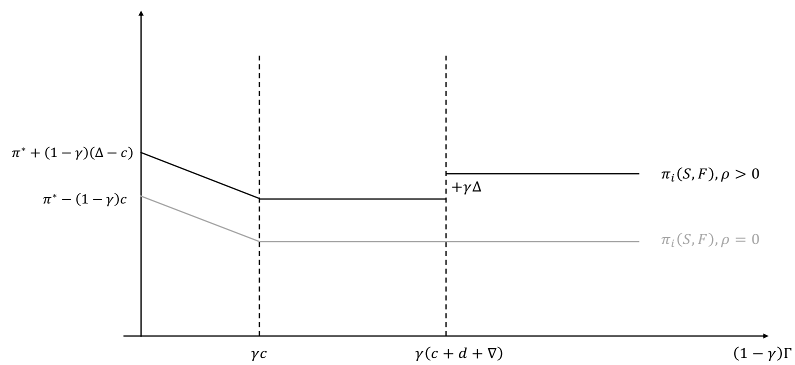

Figure 1: Representation of $\pi_i(S,F)$ for the cases $\rho > 0$ and $\rho = 0$.

6. Optimal investments

Perfectly anticipating the future stages, the firms simultaneously and independently set their investments in innovation. Given the investment level of its competitor, firm i chooses an investment level that solves the following maximization problem:

$$\max_{I_i} P_i\left[P_j\pi_i(S,S) + \left(1 - P_j\right)\pi_i(S,F)\right] + (1 - P_i)\left[P_j\pi_i(F,S) + \left(1 - P_j\right)\pi_i(F,F)\right] - I_i,$$

where $P_t \equiv P(\theta_t, I_t), t \in \{i,j\}$, and the profit values depend on the firms' post-innovation decisions regarding the device. The values of $\pi_i(O_1, O_2)$, where $O_i \in \{S,F\}$, are evaluated at the equilibrium and expressed in Tables 3 and 4. Differentiating the above expression results in the following first-order condition:

$$\frac{\partial P_i}{\partial I_i}\left[\left(\pi_i(S,F) - \pi_i(F,F)\right) - P_j\left(\left(\pi_i(S,F) - \pi_i(F,F)\right) - \left(\pi_i(S,S) - \pi_i(F,S)\right)\right)\right] - 1 = 0.$$

Let $A = \left(\pi_i(S,F) - \pi_i(F,F)\right)$ represent what firm i's gains by being the only successful innovator in the market. It's the difference between the profit when the firm succeeds in innovation alone and when neither firm innovates successfully.

Let $B = \left(\pi_i(S,F) - \pi_i(F,F)\right) - \left(\pi_i(S,S) - \pi_i(F,S)\right)$ measure the extra benefit a firm receives from being the sole successful innovator compared to a scenario where both firms successfully innovate. It compares the advantage of being the only successful innovator to the advantage of being one of two successful innovators. These additional benefits only arise if $\rho \neq 0$ and there is a possibility of gaining a marginal cost advantage. The first-order condition can be rewritten as:

$$\frac{\partial P_i}{\partial I_i}\left[A - BP_j\right] - 1 = 0, \tag{5}$$

where the values for A and B are provided in Table 5.

$(1-\gamma)\Gamma \downarrow$	A	B
$[0, \gamma c[$	$(1-\gamma)(d+\Delta)$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma c, \gamma(c+d+\nabla)[$	$(1-\gamma)(d+\Delta+\Gamma)-\gamma c$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$d+\Delta+(1-q^*)\gamma\nabla$	$(\Delta-\nabla)+(1-q^*)\gamma\nabla$
$[\gamma(c+d+\Delta), +\infty[$	$d+\Delta$	$(\Delta-\nabla)$

Table 5: Values of A and B for all possible values of $(1-\gamma)\Gamma$. The value of q^* is given by equation (4).

It is evident that $A > B \geq 0$, indicating that the firm gains more by being the sole successful innovator. In the Appendix B, we demonstrate that each firm's profits are concave with respect to its own investment level, ensuring a unique equilibrium. Lemma 2 explains how the analysis of the optimal investment strategy hinges upon the value for ρ.

Lemma 2: *If $\rho = 0$, meaning that the innovation can only reduce fixed compliance costs, a dominant strategy equilibrium emerges: each firm's optimal level of investment depends only on whether it activates the device or not. If $\rho > 0$, meaning that the innovation may allow one firm to gain a marginal cost advantage, the investment levels become strategic substitutes.*
Proof: See Appendix B.

The possibility to exhibit a marginal cost advantage makes the investment decisions strategically interlinked, prompting each firm to reduce its investment when the rival firm increases its own. This behavior increases the likelihood that the rival firm will gain a cost advantage. As we will observe, this negative impact on the incentive to innovate results in ambiguous comparative statics with respect to certain parameters. To clarify the analysis, we examine the cases $\rho = 0$ and $\rho > 0$ separately.

6.1. Strategically independent investment decisions ($\boldsymbol{\rho = 0}$).

If $\rho = 0$, the optimal investment level satisfies the following equation:

$$A' \frac{\partial P_i}{\partial I_i} - 1 = 0, \tag{6}$$

where the values of A' are provided in the Table 6 below.

Range for $(1-\gamma)\Gamma \downarrow$	Cheating strategy	A'
$[0, \gamma c[$	ON	$(1-\gamma)d$
$[\gamma c, \gamma(c+d)[$	ON when innovation fails	$(1-\gamma)(d+\Gamma) - \gamma c$
$[\gamma(c+d), +\infty[$	OFF	d

Table 6: Values of A' for all possible values of $(1-\gamma)\Gamma$.

As $(1-\gamma)\Gamma$ increases, the value of A' also increases (i.e., the investment level rises). Thus, as one may expect, Table 6 points to the fact that the decisions to activate the device and invest in innovation are strategic substitutes.

Proposition 2: *If $\rho = 0$, meaning that the innovation can only reduce the fixed compliance costs, the optimal investment is continuous with respect to all exogenous parameters. It is non-increasing in c and γ, non-decreasing in* Γ, *and increasing in θ_i and d.*
Proof: See Appendix.

Everything else being equal, an increase in the penalty Γ has no impact on investment if $(1-\gamma)\Gamma < \gamma c$ or if $(1-\gamma)\Gamma \geq \gamma(c+d)$. In the former case, it is because the payment of the fine is not dependent on innovation outcome, and in the latter, it is because the firm does not activate the device. Between these two values, the penalty fee reduces the profits the firm earns when it fails to innovate, as it activates the device and may pay the fine. Hence, an increase in the penalty stimulates investment.

An increase in θ_i enhances the effectiveness of the investment in achieving success, which incentivizes firms to increase their investments. Similarly, as the effectiveness of the device improves (γ increases), firms become more inclined to rely on it for compliance, thereby reducing their investments.

The parameter c impacts investment only if $(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d)[$. If $(1-\gamma)\Gamma < \gamma c$, the firm activates the device regardless of the innovation outcome. Consequently, regardless of whether it innovates, it must pay the fixed cost c whenever the device is detected. If $(1-\gamma)\Gamma \geq \gamma(c+d)$, the firm never uses the device. In this case, regardless of whether it innovates successfully, it must always pay the fixed cost c. However, if $(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d)[$, the decision to activate the device is made only when innovation fails. Therefore, as c increases, the firm becomes more inclined to rely on the device to avoid incurring the higher cost c. Consequently, the prospect of not investing in innovation becomes more appealing as c rises. Hence, an increase in c discourages investment within this interval. Finally, the fixed cost d can be eliminated either through cheating or innovation. Hence, an increase in d could incentivize the firm to cheat (thereby reducing its investment) or to innovate (thereby increasing its investment). A successful innovation eliminates the fixed cost d with certainty, whereas activating the device only eliminates this cost if the device is undetected. Overall, investment in innovation increases with d, and the impact of d is more pronounced when the firm does not intend to rely on the device.

We can now summarize the outcome of the entire game considering $\rho = 0$. In this case, each firm relies on a dominant strategy for both its investment decision and its decision to activate the device. There is no strategic interaction between the two firms prior to market competition. At T=3, if $(1-\gamma)\Gamma < \gamma c$ the firms activate the device even if innovation succeeds. If $(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d)[$, each firm activates the device only when it fails to innovate. Finally, they never activate the device if $(1-\gamma)\Gamma \geq \gamma(c+d)$. The level of investment is non-decreasing with $(1-\gamma)\Gamma$ indicating that the firms increasingly rely on innovation as the regulatory agency improves its ability to detect and penalize cheating.

6.2. Strategically interlinked investment decisions ($\boldsymbol{\rho > 0}$).

As shown in the Appendix B, if $\rho > 0$, the investment decisions become strategically dependent, and they exhibit a discontinuity as $\pi_i(S, F)$ is discontinuous. This, in turn, leads to a discontinuity in the optimal level of investment, as explained in Lemma 3.

Lemma 3: *If* $(1-\gamma)\Gamma < \gamma(c + d + \nabla)$ *or if* $(1-\gamma)\Gamma > \gamma(c + d + \nabla)$, *the optimal investments are continuous with respect to all exogenous parameters. However, there is a discontinuity in the optimal level of investment at* $(1-\gamma)\Gamma = \gamma(c + d + \nabla)$. *At this threshold, everything else being equal, and for any given investment level set by the rival, firm i's best reply shifts upwards*.

Proof: It is straightforward to demonstrate that the values for A and B increase continuously with $(1-\gamma)\Gamma$ if $(1-\gamma)\Gamma < \gamma(c + d + \nabla)$ and if $(1-\gamma)\Gamma > \gamma(c + d + \nabla)$. At $(1-\gamma)\Gamma = \gamma(c + d + \nabla)$, the profits $\pi_i(S, F)$ are discontinuous, as the value for $\pi_i(S, F)$ shifts upward by $\gamma\Delta$. Regarding the optimal investment, at $(1-\gamma)\Gamma = \gamma(c + d + \nabla)$, the values for A and B shift upward, as the value of $[A - BP_j]$ increases by $\gamma\Delta(1 - P_j) > 0$. This indicates that the firm becomes more inclined to invest as $(1-\gamma)\Gamma$ surpasses the threshold $\gamma(c + d + \nabla)$.

To simplify the equilibrium analysis, we consider symmetric firms so that $\theta_1 = \theta_2 = \theta$ and conduct some comparative statics presented in Lemma 4. Since the exogenous parameters are identical for both firms, we can focus on the symmetric equilibrium where $I_1 = I_2 = I^*$,where $I^*$is the solution to the following identity:

$$\left.\frac{\partial P}{\partial I}\right|_{I^*} [A - BP(\theta, I^*)] - 1 \equiv 0. \tag{7}$$

Lemma 4: *If* $\theta_1 = \theta_2 = \theta$, *and for any* $(1-\gamma)\Gamma < \gamma(c + d + \nabla)$ *and* $(1-\gamma)\Gamma > \gamma(c + d + \nabla)$, *the optimal investment level is continuous with respect to all exogenous parameters.*

The optimal symmetric investment level increases with d, Δ, ∇, *and* ρ. *However, the comparative statics with respect to* c, γ, *and* Γ *are not monotone. Finally, the overall impact of an increase in* θ *is indeterminate.*

Proof: See Appendix B.

The impact of each parameter $x \in \{c, d, \Gamma, \rho, \gamma, \Delta, \nabla\}$ (i.e. all parameters other than θ) on the investment level depends entirely on how it affects the values of A and B. In contrast, changes in θ do not influence A or B but instead affect investment through their impact on the probability of success.

The comparative statics with respect to θ should not be interpreted as an evaluation of whether a firm invests more as its own innovative capability improves. From the first-order condition given by equation (5), it is evident that firm i's optimal investment level rises with θ_i, given that the objective function is concave in I_i and that $P_{\theta I} > 0$. Rather, $\frac{dI^*}{d\theta}$ assesses how the optimal symmetric investment levels change as both firms become better innovators. As their own innovative capability increases, each firm has an incentive to increase its investment. However, because investments are strategic substitutes, firms are tempted to reduce their own investment in response to their rival's increased investment. Figure 2 illustrates a situation where the equilibrium investment level decreases as θ increases because firms respond aggressively to their opponent's investment decision. It is worth noting that a reduction in investment does not necessarily lead to a decline in the probability of success. As firms become more capable to innovate, their probability of successful innovation increases for any given level of investment.

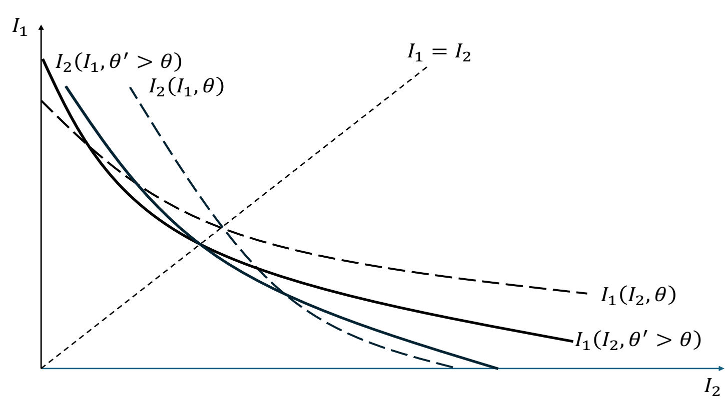

Figure 2: Dashed lines represent the best-reply functions assuming $\theta_1 = \theta_2 = \theta$. Plain lines represent the best-reply functions assuming $\theta_1 = \theta_2 = \theta' > \theta$.

For any exogenous parameters other than θ and ∇, i.e. $x \in \{c, d, \Gamma, \rho, \gamma, \Delta\}$, we always have either $\frac{dA}{dx} \geq 0$ and $\frac{dB}{dx} \geq 0$, or $\frac{dA}{dx} \leq 0$ and $\frac{dB}{dx} \leq 0$. It is never the case that one of these parameters positively impacts A while negatively impacting B, or vice-versa. This indicates that any increment in these parameters activates two *opposing* incentives. An increase in A leads to a greater incentive to innovate. Conversely, an increase in B acts as a deterrent, as it implies that investments become stronger strategic substitutes. In other words, a marginal increase in I_j leads firm i to decrease its own investment more aggressively as B increases. In all cases we show that the impact on A dominates: the incentive to increase investments to reduce fixed costs and gain a cost advantage always prevails. Therefore, when the comparative statics are non-monotonic, it is never due to conflicting effects. As we shall see, the impact of c, γ and Γ shifts from being positive to negative, or vice-versa. The reason why this occurs is that each of the exogenous parameters not only affects the profits that the firms earn in the last stage but also influences their decision to cheat. Within the range $(1-\gamma)\Gamma \in [\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$, firms cheat with a probability of $(1-q^*)$ when both fail to innovate. A change in any exogenous parameters affects the weight placed on activating the device, which can alter the impact that some parameters have on investment.

The parameter ∇ is the only one that does not necessarily activate opposing incentives. Specifically, while $\frac{dA}{d\nabla} \geq 0$, we may either have $\frac{dB}{d\nabla} \geq 0$ or $\frac{dB}{d\nabla} < 0$. Nevertheless, the impact of ∇ on investment is always unambiguous.

- **Parameters that stimulate innovation investments: $\boldsymbol{d}, \boldsymbol{\rho}, \Delta$ and ∇.**

The parameters d and Δ have a non-negative impact on both A and B, meaning that they activate the two opposing incentives mentioned earlier. However, an increase in d or Δ always leads to an increase in investment. In terms of ∇ we establish the following:

- For any $(1-\gamma)\Gamma \notin]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, this parameter has a no effect on A, but it incentivizes firms to increase their investment by reducing the extent to which investments are strategic substitutes (i.e., B decreases).

- For any $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, an increase in ∇ leads both firms to assign a higher weight to using the defeat device. As a result, the value of A increases, while the value for B may either increase (if q^* is low enough) or decrease (if q^* is high enough). The positive impact on A prevails and is further emphasized if q^* is sufficiently high.

Finally, ρ, which influences investment through its positive effect on both Δ and ∇, has a positive impact on the investment, as illustrated in Figure 3. The comparative statics with respect to the parameter t are presented in detail in Section 7 as they are non-trivial.

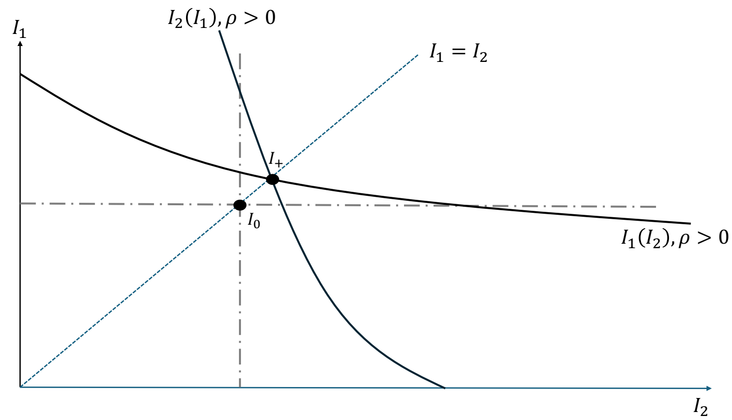

Figure 3: Best-reply functions for the cases $\rho = 0$ (dashed lines) and $\rho > 0$ (plain line). The point I_0 is the equilibrium if $\rho = 0$ and I_+ is the equilibrium if $\rho > 0$.

- **Parameters with a non-monotonic impact on innovation investments: c, γ, Γ.**

Let us first consider any $(1-\gamma)\Gamma \notin]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, representing all intervals for $(1-\gamma)\Gamma$ where firms do not rely on a mixed strategy when they both fail to innovate. In such cases, the comparative statics with respect to c, γ and Γ are straightforward. An increase in c or γ tends to reduce investments in innovation, whereas an increase in the fee Γ generally encourages such investments. More precisely, we can summarize the comparative statics as follows:

- If $(1-\gamma)\Gamma \geq \gamma(c+d+\Delta)$, none of these parameters influence the investment decision, as the firm chooses not to use the defeat device. Consequently, γ and Γ become irrelevant while c is consistently incurred.
- At the opposite extreme, if $(1-\gamma)\Gamma < \gamma c$, the parameters c and Γ have no impact on the investment decision because the firm incurs these costs whenever the device is detected, regardless of innovation outcome. However, the parameter γ negatively affects the innovation investment because the firm relies on the device regardless of its innovation outcome and invests less as the device's effectiveness improves.
- Finally, if $(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d+\nabla)[$, the parameters c and γ have a negative impact on innovation investments, while a higher penalty fee has a positive impact. Within this range, one can argue that failing to innovate becomes a way to avoid the compliance cost c, as the firm uses the device only if it fails to innovate. Therefore, as c or γ increases, innovation becomes less appealing. Conversely, as the penalty fee increases, firms are more motivated to avoid it and focus on innovation.

If $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, the comparative statics' outcomes are reversed. Within that interval, these parameters influence the firms' behavior when they both fail to innovate and approach cheating using a mixed strategy equilibrium. Increases in c and in γ prompt firms to place more weight on activating the device. This decision reduces the value of $\pi_i(F,F), i=1,2$, causing the values of A and B to increase with c and γ. The positive impact on A prevails, leading firms to invest more as c and γ increase. Thus, firms invest more in innovation even when they can rely on more effective defeat devices. Conversely, an increase in Γ causes firms to place less weight on activating the device, which raises the value of $\pi_i(F,F), i=1,2$. As a result, the values of A and B decrease with the penalty fee within this range. The negative impact on A prevails, leading firms to invest less in innovation as the penalty fee increases.

7. Analysis and policy implications

In this section, we investigate further the firms' equilibrium behavior, as documented in previous sections, and derive some policy implications. Based on the analysis of Proposition 1 and Lemma 4, we reach the following conclusions.

- **In what environment will firms be more likely to cheat?**

We establish that firms are more likely to activate the defeat device once innovation has failed (see Table 2). Thus, cheating is more likely to occur when firms have a low capability to innovate successfully (due to a low θ) or when they allocate minimal resources to R&D. In that respect, a lower reliance on cheating could be achieved lowering the cost of R&D investments. Table 2 also shows that high compliance costs (associated with stringent standards) should be accompanied by increased enforcement efforts to ensure true compliance. This result is in line with Hu et al. (2021). In the Volkswagen case, cheating may have resulted from the combination of high compliance costs needed to meet stringent emission standards for diesel vehicles in the U.S. and a belief that the defeat devices would be very difficult to detect. In fact, the device was accidentally discovered, not by regulators but by scientists pursuing unrelated research objectives.[14]

- **Does the prospect of activating a defeat device deter investment in innovation?**

To answer this question, let us assume that the enforcement efforts, captured by $(1-\gamma)\Gamma$, increase while all other parameters remain constant.[15] As $(1-\gamma)\Gamma$ increases, the firm becomes less likely to activate the device after innovation. Given this anticipation, how does the investment level change? The answer is illustrated in Table 7 and Figure 4 below.

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[14] The ICCT, who commissioned the study, thought they could prove that these vehicles were genuinely clean and that modern diesels could therefore make a significant contribution to improving both CO_2 emissions and air quality in future transport policy. Available at : https://theconversation.com/how-volkswagen-got-caught-cheating-emissions-tests-by-a-clean-air-ngo-47951.

[15] We established that what matters is the size of $(1-\gamma)\Gamma$ relative to the compliance costs.

Region for $(1-\gamma)\Gamma$	$0 \to \gamma c$	$\gamma c \to \gamma(c+d+\nabla)$	$\gamma(c+d+\nabla) \to \gamma(c+d+\Delta)$	$\gamma(c+d+\Delta) \to +\infty$
Cheating device	ON	ON when innovation fails	OFF Mixed eq. when both fail	OFF
Impact of $(1-\gamma)\Gamma$ on I	Positive	Positive	Negative	None
Investment strategy	The investment increases over that range.		Investment jumps at $\gamma(c+d+\nabla)$ and then decreases	Investment is fixed and is higher than at $\gamma(c+d+\nabla)$.

Table 7: Comparative statics of the penalty fee $(1-\gamma)\Gamma$ on the optimal investment in innovation for different regions of $(1-\gamma)\Gamma$.

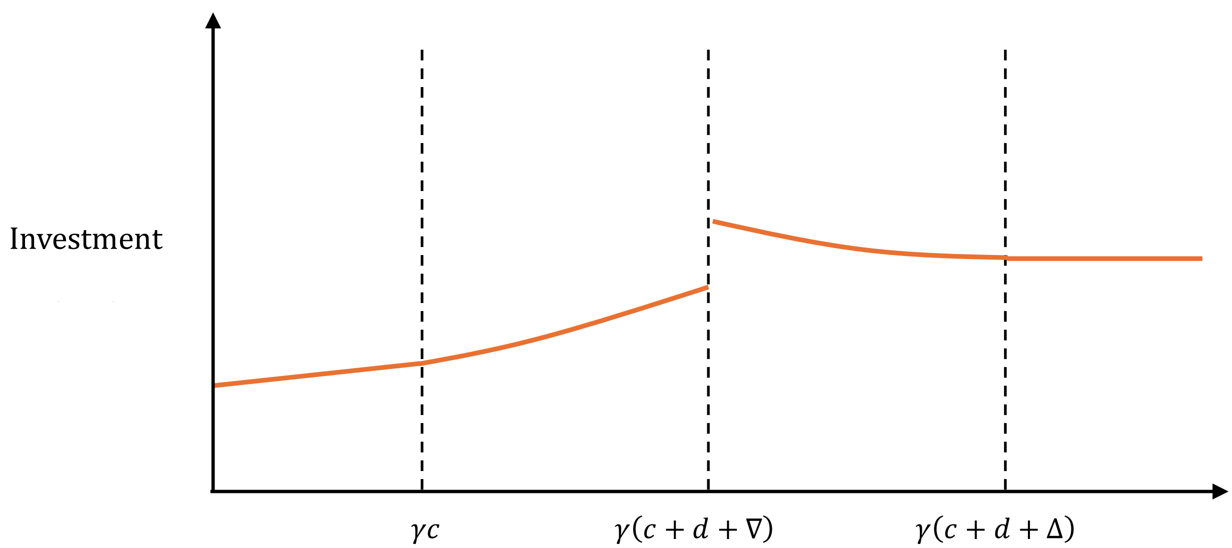

Figure 4: Representation of the optimal investment in innovation as $(1-\gamma)\Gamma$ increases.

Investments are higher when $(1-\gamma)\Gamma \geq \gamma(c+d+\nabla)$ compared to when $(1-\gamma)\Gamma < \gamma(c+d+\nabla)$. This indicates that, *overall*, firms invest more as they anticipate relying less on the device. This result suggests that the prospect of cheating acts as a deterrent to innovation investment, making the decisions to cheat and innovate strategic substitutes. However, for all $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, the comparative statics on investment yield counter-intuitive results. Specifically, we show that an increase in $(1-\gamma)\Gamma$ (stronger enforcement) reduces the likelihood of activating the device, as one would expect. However, this also increases the value of profits when the firms fail to innovate, making failure in innovation less costly. As a result, firms invest less in innovation, increasing the likelihood the innovation failure, and consequently, their reliance on cheating. Conversely, weaker enforcement efforts incentivize investment in innovation, making firms more likely to succeed and reducing their reliance on cheating.

- **What impact does competition have? A focus on the parameter t.**

The greater t is, the more differentiated the automobiles become, leading to reduced competition between firms. If investment decisions are strategically independent (i.e. $\rho = 0$), t has no impact on firms' decisions. However, if investment decisions are strategically interlinked (i.e. $\rho > 0$), an increase in t influences cheating and innovation investment through Δ and ∇. Specifically, increased competition, represented by a lower t, leads to a higher Δ and a lower ∇. Since increases in both parameters stimulate investment, the overall impact of t on I^* is ambiguous. Indeed, the comparative statics with respect to t are such that

$$\text{sign of } \frac{dI^*}{dt} = \text{sign of } \left[\frac{dA}{dt} - \frac{dB}{dt}P^*\right].$$

where A and B are given in Table 5. [16]

An increase in product differentiation triggers two *opposing* forces. First, an increase in t decreases A, indicating that firms have less to gain from achieving a cost advantage, which deters them from investing in innovation. At the same time, it also decreases B, meaning that

[16] See proof of Lemma 4 in Appendix B.

each firm responds less aggressively to its rival's increased investment, which would stimulate investments. Unlike previous comparative statics, it is not clear which effect dominates, and the overall impact depends on the firms' strength as innovators. Indeed, for any $(1-\gamma)\Gamma \notin]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, the impact of t discussed above leads to the following outcome:

$$\text{sign of } \frac{dI^*}{dt} = \text{sign of } [2P^* - 1].$$

When firms are likely to innovate (e.g. when θ is high, such that $2P(\theta, I^*) - 1 > 0$), an increase in product differentiation leads to higher investments in innovation. Greater confidence in their capability to innovate successfully encourages firms to focus on innovation. As product differentiation increases, this effect is amplified because each firm responds less aggressively to its rival's investment decisions. By contrast, when firms are less likely to innovate (e.g., when θ is low, such that $2P(\theta, I^*) - 1 < 0$), greater product differentiation leads to lower investments in innovation. Aware of their low capability to innovate, firms invest less in R&D. Furthermore, as product differentiation increases, the firms' incentives are now driven by the fact that they have less to gain from achieving a cost advantage, further reducing their investments. In other words, firms with low innovation capability are encouraged to invest in innovation only when market competition intensifies.

In terms of cheating, and since innovation and cheating are strategic substitutes, we can conclude that firms with a high capability to innovate successfully will focus more on innovation, and less on cheating, as in less competitive markets. Indeed, an increase in competition will deter firms with a high capability to innovate to invest in innovation but incentivize weak innovators to do so. Thus, as product differentiation increases, strong innovators invest more and are then less likely to rely on a cheating device. In contrast, weak innovators are less likely to rely on a defeat device to address compliance when market competition increases. It is therefore possible to speculate that Volkswagen may not have been particularly capable of successful innovation given the abatement technology that it relied on, Lean NOx traps (LNT), compared to its competitors who developed Selective Catalytic Reduction (SCR). Additionally, as a company offering diesel vehicles in a country where petrol is generally the preferred option,[17] Volkswagen faced high product differentiation relative to its competitors. As the above shows, weak innovators are more likely to reduce their investment and rely on a defeat device when product differentiation is high.

If $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, changes in t simultaneously affect both the firm's investment and the likelihood of cheating. This simultaneity complicates the analysis of the comparative statics. Although this range becomes narrower as t increases, examining some of the comparative statics is insightful, as it reveals how market competition and enforcement efforts act as strategic substitutes. Within the range $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$, one can verify that the impact of t on q^* is not monotonic:

$$\text{sign of } \frac{dq^*}{dt} = \text{sign of } \left[(1-\gamma)\Gamma - (c+d) - \frac{\gamma\rho}{3}\right],$$

$$\text{and } \frac{dq^*}{dt} = \begin{cases} -\dfrac{1}{2t} \text{ at } (1-\gamma)\Gamma = \gamma(c+d+\nabla), \\ +\dfrac{1}{2t} \text{ at } (1-\gamma)\Gamma = \gamma(c+d+\Delta). \end{cases}$$

If $(1-\gamma)\Gamma \in \left]\gamma(c+d+\nabla), (c+d)+\frac{\gamma\rho}{3}\right]$, an increase in product differentiation leads the firms to place greater emphasis on activating the device. If $(1-\gamma)\Gamma \in \left](c+d)+\frac{\gamma\rho}{3}, \gamma(c+d+\Delta)\right]$, an increase in product differentiation leads the firms to be less likely to activate the device. When the enforcement efforts are low, firms are more inclined to cheat because the weight put on "OFF" is low. However, this weight increases as competition intensifies, and products become more homogeneous. As a result, firms become less willing to cheat as market competition increases. By opposition, when enforcement efforts are high, the weight placed on "OFF" is initially higher, but it decreases as products become more homogeneous. In this case, firms are less willing to cheat as market competition decreases.

[17] Volkswagen promoted their diesel models on their characteristic of better fuel economy, while still meeting low NOx emissions standards. The abatement technology that they installed reduced engines' fuel efficiency when operating correctly.

If we considering the post-innovation cheating decision more broadly, we observe that a lower t causes the range $(1-\gamma)\Gamma \in]\gamma(c+d+\nabla), \gamma(c+d+\Delta)]$ to widen. This implies that as competition increases, enforcement efforts do no need to be as strong to deter cheating. These findings suggest that market competition and regulator's enforcement efforts act as substitutes in deterring the firms' cheating behavior.

- **How valuable is the ability to use a defeat device?**

As automobiles become increasingly computerized, the potential for cheating also increases. Do firms benefit from their easier access to an alternative compliance approach? Clearly, if the defeat device is easily detectable or if the penalty for cheating is sufficiently high—specifically if $(1-\gamma)\Gamma \geq \gamma(c+d+\Delta)$— cheating has no value as the firms would never activate the device. However, in all other cases, firms exploit their ability to activate the device to deceive regulators. To explore whether firms would be better off if the possibility to cheat was eliminated., we consider a setting where firms can credibly commit not to cheat (by signing a voluntary agreement), and that this commitment is verifiable. Considering symmetric firms, where $\theta_1 = \theta_2 = \theta$, let Π^C denote a firm's profits when both firms commit not to cheat and let Π^* denote the profits a firm earns when neither firm commits. Additionally, let $\widehat{\Pi}^C$ represent a firm's profits when it commits not to cheat while its rival does not, and let $\widehat{\Pi}^*$ denote the profits when it does not commit, while its rival does. Table 8 summarizes all possible outcomes in terms of the profits earned by each firm in each scenario. The specific expressions for these profits and their calculations can be found in Appendix B.

	Firm 2 commits	Firm 2 does not commit
Firm 1 commits	Π^C, Π^C	$\widehat{\Pi}^C, \widehat{\Pi}^*$
Firm 1 does not commit	$\widehat{\Pi}^*, \widehat{\Pi}^C$	Π^*, Π^*

Table 8: Profits earned by Firm 1 and Firm 2, respectively, in all possible scenarios.

To simplify the analysis, we assume the probability of success follows a specific functional form:

$$P(I,\theta) = \frac{\theta I}{(1+\theta)(1+I)}. \tag{8}$$

- **Case $\boldsymbol{\rho = 0}$**

If $\rho = 0$, we have $\Pi^* = \widehat{\Pi}^*$ and $\Pi^C = \widehat{\Pi}^C$. In this case, the optimal investment levels when firms do not commit (I^*) and when they commit not to activate the device (I^C) are defined by the following equations:

$$(1+I^*)^2 = \frac{\theta A'}{(1+\theta)} \text{ and } (1+I^C)^2 = \frac{\theta d}{(1+\theta)}, \tag{9}$$

where the value of A' is given in Table 6.

Lemma 5: *If $\rho = 0$, and the probability of success is given by equation (8), firms are always better off when they do not commit. The gap between Π^* and Π^C decreases as the firms' capability to innovate increases.*

Proof: See Appendix B.

As shown in the Appendix B, if $\rho = 0$, the earned profits are given as follow:

$$\Pi^* = \pi^* - (1-\gamma)(c+d+\Gamma) + A'P(\theta, I^*) - I^*,$$
$$\Pi^C = \pi^* - c - d\big(1 - P(\theta, I^C)\big) - I^C,$$

where A' is given in Table 6. Additionally, we have

$$\frac{d\Pi^*}{d\theta} = A'P_\theta|_{I=I^*} \text{ and } \frac{d\Pi^C}{d\theta} = dP_\theta|_{I=I^C}.$$

Given assumption (v) on the probability function, and since $A' \leq d$ and $I^* \leq I^C$, we can conclude that as θ increases, the profits under commitment grow faster than those earned under the possibility of cheating. However, this result does not imply that the two profit functions intersect. In the Appendix B, we demonstrate that, with the chosen probability function in equation (8), there is an asymptotic convergence, and we always have $(\Pi^* - \Pi^C) \geq 0$.

- **Case $\rho > 0$**

Even when $\theta_1 = \theta_2 = \theta$, the analysis here is quite complex. Using equations (7) and (8), the optimal investment levels, when firms do not commit (I^*) and when they commit not to activate the device (I^C), are defined by the following equations:

$$(1+I^*)^2 = \frac{\theta}{(1+\theta)}\big(A - BP(I^*,\theta)\big),$$
$$(1+I^C)^2 = \frac{\theta}{(1+\theta)}\big(d + \Delta - (\Delta - \nabla)P(I^*,\theta)\big), \tag{10}$$

where the values for A and B are given in Table 5.

Lemma 4 highlights the fact that the comparative statics for the optimal investment with respect to θ are not straightforward. However, we can evaluate the value of $(\Pi^* - \Pi^C)$ at $\theta = 0$ to gain insight into whether commitment may be more valuable when a firm stands to gain market share.

By assumption, $P(0, I^*) = P(0, I^C) = 0$, and we have $I^C = I^* = 0$ at $\theta = 0$.

Range for $(1-\gamma)\Gamma$	Profits $(\Pi^* - \Pi^C)$ at $\theta = 0$
$[0, \gamma c[$	$\gamma(c+d) - (1-\gamma)\Gamma \geq 0$
$[\gamma c, \gamma(c+d+\nabla)[$	$\gamma(c+d) - (1-\gamma)\Gamma$ Positive at $(1-\gamma)\Gamma = \gamma c$ Negative at $(1-\gamma)\Gamma = \gamma(c+d+\nabla)$
$[\gamma(c+d+\nabla), \gamma\ (c+d+\Delta)[$	$-(1-q^*)\gamma\nabla \leq 0$

Table 9: Value of $(\Pi^* - \Pi^C)$ at $\theta = 0$ with $\rho > 0$.

It is now possible for $(\Pi^* - \Pi^C) < 0$ at $\theta = 0$, meaning that firms can be better off when they commit not to cheat, even if they are unable to innovate. To analyze what happens in equilibrium for $\theta > 0$, we use Mathematica to calculate the equilibrium profits in the game, as shown in Table 8. We focus on the interval where the sign of $(\Pi^* - \Pi^C)$ changes, specifically $[\gamma c, \gamma(c+d+\nabla)[$. Tables 10 to 12 represent the outcomes of the game considering the following exogenous parameters: $\theta = 4, t = 20, \rho = 1, d = 3, c = 1, \gamma = 0.3$, with the penalty fee Γ increasing.

In Tables 10, 11 and 12 we highlight the equilibrium profits in bold. We show that it is always a dominant strategy for firms not to commit. In Table 10 this outcome is efficient (it maximizes the firms' individual and overall profits). Table 11 shows that firms could be almost indifferent between committing and not committing to cheat. Finally, Table 12 shows that retaining the ability to cheat leads firms into a prisoner's dilemma, where both would be better off committing not to cheat.

	Firm 2 commits	Firm 2 does not commit
Firm 1 commits	6.3, 6.3	6.24, 6.73
Firm 1 does not commit	6.73, 6.24	**6.67, 6.67**

Table 10: Equilibrium profits if $\Gamma = 1$.

	Firm 2 commits	Firm 2 does not commit
Firm 1 commits	6.298, 6.298	6.231, 6.365
Firm 1 does not commit	6.365, 6.231	**6.297, 6.297**

Table 11: Equilibrium profits if $\Gamma = 1.72$.

	Firm 2 commits	Firm 2 does not commit
Firm 1 commits	6.298, 6.298	6.229, 6.302
Firm 1 does not commit	6.302, 6.229	**6.232, 6.232**

Table 12: Equilibrium profits if $\Gamma = 1.85$.

To summarize: the possibility to rely on a cheating device is a strategy that the firms hold on to when the enforcement efforts are low. In such cases, the firms activate the device either systematically or when they fail to innovate and the decision not to commit is efficient. As enforcement efforts increase so that the decision to activate the device is only taken when innovation fails, the decision not to commit becomes inefficient. This means that firms hold on to the possibility of cheating more often than they should. Finally, as the enforcement efforts increase even further whether the firms commit not to cheat is irrelevant as the possibility to activate the device has no value for the firm.[18]

Since the Volkswagen scandal, many other automobile firms have been caught cheating in emissions tests carried out by NGOs and government agencies, demonstrating that firms held up to the possibility of cheating (Meyer et al., 2023). Further evidence is found in the voluntary agreement signed by the European Commission with the automobile industry to collectively reduce CO2 emissions by 25%, which was not achieved and was replaced in 2008 by a regulation.[19]

Considering further simulations letting θ increases, we can establish the following:

- If $(1-\gamma)\Gamma < \gamma(c+d)$, we always have $(\Pi^{*} - \Pi^{C}) > 0$ as θ increases, and the outcome resembles that of Table 10: retaining the cheating device is a dominant strategy and represents the efficient outcome.
- If $(1-\gamma)\Gamma > \gamma(c+d)$, we always have $(\Pi^{*} - \Pi^{C}) < 0$ as θ increases, and the outcome resembles that of Table 12: retaining the cheating device remains a dominant strategy, but it is not an efficient outcome.

8. Conclusions

This paper examines how automobile manufacturers approach the requirement to comply with new emission standards focusing on investment in innovation and cheating as the primary compliance strategies. We consider a setting where, when successful, innovation can achieve compliance at lower costs while cheating, when it is not detected, signals compliance while eliminating all compliance costs. The underlying assumption is that firms prioritize the option of developing a truly compliant technology but may decide to activate a

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[18] When they rely on a mixed strategy equilibrium the firms are effectively indifferent between cheating and not doing so.

[19] Transport and Environment (2007) *Carmakers could face legislation on climate,* Press Release. https://www.transportenvironment.org/articles/carmakers-could-face-legislation-climate

cheating device based on the outcome of innovation, the monitoring system in place and the size of the compliance costs.

We show that, post-innovation, the decision to cheat is based on a cost benefit analysis whereby firms weigh the potential savings from avoiding compliance costs against the risk of detection and associated penalties. We also find that firms are more likely to activate the cheating device when they fail to innovate. Hans-Dieter Pötsch, chairman of Volkswagen's supervisory board, provides some evidence for this possibility, stating that *'the company's engineers decided to cheat on emissions tests in 2005 because they couldn't find a technical solution within the company's time frame and budget to build diesel engines that would meet U.S. emissions standards*' (Goodman, 2015). We conclude that firms are more likely to cheat when they have a low capability to innovate successfully, or when they choose to devote little resources to R&D. Furthermore, we establish that investment in innovation and cheating are strategic substitutes. Firms invest less in innovation when they expect to use cheating device, either systematically or as a fallback when innovation fails.

The types of fixed compliance costs – unavoidable and avoidable –have differing impacts on the investment decision. The unavoidable fixed compliance cost can either stimulate or deter firms from investing in innovation depending on the level of regulator's enforcement efforts. The avoidable fixed compliance cost, on the other hand, always promotes innovation investment. This finding provides further understanding of the impact of environmental regulations on innovation (Porter, 1996, Jaffe and Palmer, 1997, and Jaffe et al., 2002). In the automobile sector, regulations tend to spur an "ecology of innovation" (Bresnahan and Yao, 1985, Lee et al., 2010, and Lee et al., 2011), suggesting that large compliance costs can be avoided subject to innovation. More flexible policy instruments such as attribute-based standards or targets based on the manufacturer fleet average provide more scope for lower unavoidable costs and higher avoidable costs and can therefore drive innovation as a compliance strategy.

While we show that competition can reduce the need for monitoring to induce true compliance, we also establish that it has a non-obvious impact on the investment strategy. As products become closer substitutes, firms with a high capability to innovate lower their investments in innovation while weak innovators increase their investment. Finally, we evaluate whether the increased ease of cheating benefits the automobile industry. We consider this in the context of firms having the option to commit to honesty, such as through voluntary agreements. The main finding is that firms tend to hold on to the possibility to cheat even when it is inefficient.

The paper's overall findings suggest that regulators should account for the path-dependent nature of cheating and investment decisions. Closer monitoring of cheating will incentivize firms to invest in innovation and policies aiming to facilitate R&D investments will deter the firms from cheating. While our analysis is centered on the well-documented automobile industry, the model framework and key insights regarding the two interrelated compliance strategies are generalizable to other environmentally regulated industries. The findings offer valuable implications for sectors facing similar regulatory challenges and sustainability

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APPENDIX A

Parameters	Definition
$v > 0$	Value gathered by a consumer from purchasing product 1 or 2.
$t > 0$	Transport cost measuring product differentiation.
$p_i, i = 1,2$	Endogenous price for product i, $(i = 1,2)$.
$I_i, i = 1,2$	Fixed compliance cost capturing the endogenous investment in R&D.
$\theta_i, i = 1,2$	Exogenous parameter capturing firm i's capability to innovate $i \in \{1,2\}$.
$c \geq 0$	Fixed exogenous cost, incurred whether innovation fails or succeeds, to achieve true compliance.
$d \geq 0$	Fixed exogenous cost, incurred in addition to c when innovation fails, to achieve true compliance.
$\rho < 3t$	Exogenous increment in the marginal cost of production incurred when innovation fails, to achieve true compliance.
$\gamma \in]0,1]$	Effectiveness of the cheating device. The higher γ, the more difficult it is to find the device.

APPENDIX B

Proof of Lemma 1 and Proposition 1: Assume that firm i innovates successfully. At the stage 3, there are two possible strategies: each firm must decide whether to activate the defeat device (ON) or not (OFF). Let $\Pi_i(a_i, a_j)$ denote firm $i's$ profits when it adopts strategy $a_i \in \{ON, OFF\}$ while its competitor adopts $a_j \in \{ON, OFF\}$.

State $(\boldsymbol{S}, \boldsymbol{S})$: When both firms innovate successfully, neither firm can have a cost advantage, and firm i earns the following profits:

$$\Pi_i(OFF, ON) = \Pi_i(OFF, OFF) = \pi^* - c,$$

$$\Pi_i(ON, OFF) = \Pi_i(ON, ON) = \pi^* - (1-\gamma)(c + \Gamma).$$

In this situation, the strategy adopted by the rival is irrelevant. Firms 1 and 2 have a dominant strategy, which consists of activating the device if $(1-\gamma)\Gamma - \gamma c < 0$, and not activating it otherwise. This leads to two dominant strategy equilibria:

- $(1-\gamma)\Gamma < \gamma c$: Both firms activate the device.
- $\gamma c \leq (1-\gamma)\Gamma$: Neither firm activates the device.

State $(\boldsymbol{S}, \boldsymbol{F})$& $(\boldsymbol{F}, \boldsymbol{S})$: When firm i successfully innovates and firm j fails to do so, firm i can only benefit its cost advantage if its competitor either does not rely on the device or cheats and gets caught. The table below outlines the profits firm i earns based on the possible actions taken.

	$a_j = OFF$	$a_j = ON$
$a_i = OFF$	$\pi^* + \Delta - c$	$\pi^* + (1-\gamma)\Delta - c$
$a_i = ON$	$\pi^* + \Delta - (1-\gamma)(c + \Gamma)$	$\pi^* + (1-\gamma)\Delta - (1-\gamma)(c + \Gamma)$

Table B1: *Profits gathered by firm i (successful innovation)*

Regardless of its rival's strategy, firm i has a dominant strategy: activating the device if $(1-\gamma)\Gamma - \gamma c < 0$, and not doing so otherwise.

The table below shows the profits firm j earns based on the actions taken by both firms at the stage 3. If firm j does not activate the device, it faces a marginal cost disadvantage. However, if it uses the device, it can conceal this action unless it is caught.

	$a_i = OFF$	$a_i = ON$
$a_j = OFF$	$\pi^* - \nabla - (c + d)$	$\pi^* - \nabla - (c + d)$
$a_j = ON$	$\pi^* - (1-\gamma)\nabla - (1-\gamma)(c + d + \Gamma)$	$\pi^* - (1-\gamma)\nabla - (1-\gamma)(c + d + \Gamma)$

Table B2: *Profits gathered by firm j (failed innovation)*

Firm j clearly has a dominant strategy: activating the device if and only if

$$(1-\gamma)\,\Gamma - \gamma(c+d+\nabla) < 0.$$

In equilibrium, each firm employs its dominant strategy, resulting in the following dominant strategy equilibria:

- $(1-\gamma)\Gamma < \gamma c$: Both firms activate the device.
- $\gamma c \leq (1-\gamma)\Gamma < \gamma(c+d+\nabla)$: Only firm j activates the device.
- $(1-\gamma)\Gamma \geq \gamma(c+d+\nabla)$: Neither firm activates the device.

State ($\boldsymbol{F}, \boldsymbol{F}$): When both firms fail to innovate, the situation is symmetrical. The table below outlines the profits that firms i and j earn based on the actions taken by both firms at the stage 3.

	$a_j = OFF$	$a_j = ON$
$a_i = OFF$	$\pi_i = \pi_j = \pi^* - (c+d)$	$\pi_i = \pi^* - \gamma\nabla - (c+d)$ $\pi_j = \pi^* + \gamma\Delta - (1-\gamma)(c+d+\Gamma)$
$a_i = ON$	$\pi_i = \pi^* + \gamma\Delta - (1-\gamma)(c+d+\Gamma)$ $\pi_j = \pi^* - \gamma\nabla - (c+d)$	$\pi_i = \pi_j = \pi^* - (1-\gamma)(c+d+\Gamma)$

Table B3: *Profits gathered by firm i and j.*

- If $(1-\gamma)\Gamma < \gamma(c+d+\nabla)$: Activating the device is a dominant strategy equilibrium.
- If $\gamma(c+d+\nabla) \leq (1-\gamma)\Gamma < \gamma(c+d+\Delta)$: The best response to "$OFF$" is "$ON$", and the best response to "ON" is "OFF". In this scenario, there is a mixed strategy equilibrium. Assume "OFF" is played with probability q, equilibrium requires:

  $$q\big(\pi^* - (c+d)\big) + (1-q)\big(\pi^* - \gamma\nabla - (c+d)\big) =$$
  $$q\big(\pi^* + \gamma\Delta - (1-\gamma)(c+d+\Gamma)\big) + (1-q)\big(\pi^* - (1-\gamma)(c+d+\Gamma)\big).$$

  Solving for q gives:

  $$q^* = \frac{(1-\gamma)\,\Gamma - \gamma(c+d+\nabla)}{\gamma(\Delta-\nabla)}.$$

- If $(1-\gamma)\Gamma \geq \gamma(c+d+\Delta)$: Not activating the device is a dominant strategy equilibrium.

Proof of Lemma 2: The first-order condition for any interior solution requires:

$$\frac{\partial P_i}{\partial I_i}\left[P_j\big(\pi_i(S,S)-\pi_i(F,S)\big)+\big(1-P_j\big)\big(\pi_i(S,F)-\pi_i(F,F)\big)\right]-1=0.$$

Given the values of $\pi_i(O_1,O_2)$, where $O_i\in\{S,F\}$, the first-order condition can be written as

$$\frac{\partial P_i}{\partial I_i}\left[A-BP_j\right]-1=0,$$

where the values of A and B are determined by the value of $(1-\gamma)\Gamma$ and are provided in Table 5. Additionally, q^* is defined by Equation (3) in the text.

The second-order condition requires:

$$\frac{\partial^2 P_i}{\partial I_i^2}\left[A-BP_j\right]<0.$$

Given assumption (iii), which states that the success probability is concave in I_i, the first term is negative. Furthermore, it is easy to verify that under the assumption $d>\nabla$, we always have $A>B$. Therefore, we consistently find that $\left[A-BP_j\right]>0$. The objective function is concave, ensuring that the second-order condition holds and that there is a unique maximum.

If $\rho=0$ we have $\Delta=\nabla=0$, which implies $B=0$. Consequently, the optimal investment for each firm is determined solely by the solution to $A\frac{\partial P_i}{\partial I_i}-1=0$, independent of the rival's investment. Therefore, a dominant strategy equilibrium exists.

If $\rho>0$ investments are chosen strategically. By differentiating the first-order condition with respect to I_j and evaluating it at the solution, we establish that:

$$\frac{\partial^2 P_i}{\partial I_i^2}\frac{dI_i}{dI_j}-B\left(\frac{\partial P_i}{\partial I_i}\right)^2\frac{\partial P_j}{\partial I_j}=0\Rightarrow\frac{dI_i}{dI_j}<0.$$

Proof of Proposition 2: It is straightforward to observe that the variable A' is continuous, as shown by the following equations:

$$\lim_{(1-\gamma)\Gamma\to\gamma c}[(1-\gamma)(d+\Gamma)-\gamma c]=(1-\gamma)d,$$

and

$$\lim_{(1-\gamma)\Gamma\to\gamma(c+d)}[(1-\gamma)(d+\Gamma)-\gamma c]=d.$$

Since the success probability is also continuous in I, the optimal level of investment is continuous. By differentiating the first-order condition with respect to any exogenous parameters $x\in\{c,d,\gamma,\Gamma\}$, in equilibrium, we must have

$$A'\frac{\partial^2 P_i}{\partial I_i^2}\frac{dI_i}{dx}+\frac{\partial P_i}{\partial I_i}\frac{dA'}{dx}=0.$$

Given assumptions (iii) and (iv), the above equation holds if

$$\text{sign of }\frac{dI_i}{dx}=\text{sign of}\frac{dA'}{dx}.$$

We have

$$\frac{dA'}{dc}\leq 0,\frac{dA'}{dd}>0,\frac{dA'}{d\gamma}\leq 0\text{ and }\frac{dA'}{d\Gamma}\geq 0.$$

For the parameter θ_i, at the solution, we must have

$$A'\left[\frac{\partial^2 P_i}{\partial I_i^2}\frac{dI_i}{d\theta_i}+\frac{\partial^2 P_i}{\partial I_i\partial\theta_i}\right]=0.$$

Given assumptions (iii) and (v), the above equation holds if

$$\frac{dI_i}{d\theta_i}>0.$$

Proof of Lemma 4: The optimal level of investment $I^*$is the solution to the following equation:

$$\left.\frac{\partial P}{\partial I}\right|_{I^*}[A - BP(\theta, I^*)] - 1 = 0.$$

Let $P^* \equiv P(\theta, I^*)$ be the probability of success at the solution.

<u>Comparative statics with respect to θ.</u>

Notice that the values of A and B do not depend on θ. Consequently, when differentiating the first-order condition, we obtain:

$$[A - BP^*]\left[P_{II}^*\frac{dI^*}{d\theta} + P_{I\theta}^*\right] - BP_I^*\left[P_I^*\frac{dI^*}{d\theta} + P_\theta^*\right] = 0$$
$$\Longrightarrow [P_{II}^*(A - BP^*) - B(P_I^*)^2]\frac{dI^*}{d\theta} + \left(\frac{P_{I\theta}^*}{P_I^*} - BP_I^*P_\theta^*\right) = 0.$$

The first term, $[P_{II}^*(A - BP^*) - B(P_I^*)^2]$, is negative. However, the sign of the second term is not immediately obvious. Therefore, the overall effect of an increase in θ is ambiguous.

<u>Comparative statics with respect to $x \in \{c, d, \gamma, \Gamma, \Delta, \nabla\}$.</u>

Differentiating the above equation with respect to the exogenous parameters $x \in \{c, d, \gamma, \Gamma, \Delta, \nabla\}$, we obtain:

$$\left[P_{II}^*\big(A - BP(\theta, I^*)\big) - B(P_I^*)^2\right]\frac{dI^*}{dx} + P_I^*\left[\frac{dA}{dx} - \frac{dB}{dx}P^*\right] = 0.$$

Using the first-order condition and the fact that

$$\big(A - BP(\theta, I^*)\big) = \frac{1}{P_I^*},$$

we can rewrite the above as:

$$[P_{II}^* - B(P_I^*)^3]\frac{1}{P_I^*}\frac{dI^*}{dx} + P_I^*\left[\frac{dA}{dx} - \frac{dB}{dx}P^*\right] = 0.$$

Since

$$[P_{II}^* - B(P_I^*)^3]\frac{1}{P_I^*} < 0,$$

it follows that

$$\text{sign of } \frac{dI^*}{dx} = \text{sign of } \left[\frac{dA}{dx} - \frac{dB}{dx}P^*\right].$$

<u>Comparative statics with respect to c.</u>

$(1-\gamma)\Gamma$	$\frac{dA}{dc}$	$\frac{dB}{dc}$	$\left[\frac{dA}{dc} - \frac{dB}{dc}P^*\right]$
$[0, \gamma c[$	0	0	0
$[\gamma c, \gamma(c+d+\nabla)[$	$-\gamma$	0	$-\gamma$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$\frac{\gamma\nabla}{\Delta-\nabla}$	$\frac{\gamma\nabla}{\Delta-\nabla}$	$\frac{\gamma\nabla}{\Delta-\nabla}(1-P^*)$
$[\gamma(c+d+\Delta), +\infty[$	0	0	0

<u>Comparative statics with respect to d.</u>

$(1-\gamma)\Gamma$	$\frac{dA}{dd}$	$\frac{dB}{dd}$	$\left[\frac{dA}{dd} - \frac{dB}{dd}P^*\right]$
$[0, \gamma c[$	$(1-\gamma)$	0	$(1-\gamma)$
$[\gamma c, \gamma(c+d+\nabla)[$	$(1-\gamma)$	0	$(1-\gamma)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$1+\frac{\gamma\nabla}{\Delta-\nabla}$	$\frac{\gamma\nabla}{\Delta-\nabla}$	$1+\frac{\gamma\nabla}{\Delta-\nabla}(1-P^*)$
$[\gamma(c+d+\Delta), +\infty[$	1	0	1

<u>Comparative statics with respect to Γ.</u>

$(1-\gamma)\Gamma$	$\frac{dA}{d\Gamma}$	$\frac{dB}{d\Gamma}$	$\left[\frac{dA}{d\Gamma} - \frac{dB}{d\Gamma}P^*\right]$
$[0, \gamma c[$	0	0	0
$[\gamma c, \gamma(c+d+\nabla)[$	$(1-\gamma)$	0	$(1-\gamma)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$-\frac{(1-\gamma)\nabla}{\Delta-\nabla}$	$-\frac{(1-\gamma)\nabla}{\Delta-\nabla}$	$-\frac{(1-\gamma)\nabla}{\Delta-\nabla}(1-P^*)$
$[\gamma(c+d+\Delta), +\infty[$	0	0	0

Comparative statics with respect to Δ.

$(1-\gamma)\Gamma$	$\frac{dA}{d\Delta}$	$\frac{dB}{d\Delta}$	$\left[\frac{dA}{d\Delta}-\frac{dB}{d\Delta}P^*\right]$
$[0,\gamma c[$	$(1-\gamma)$	$(1-\gamma)$	$(1-\gamma)(1-P^*)$
$[\gamma c,\gamma(c+d+\nabla)[$	$(1-\gamma)$	$(1-\gamma)$	$(1-\gamma)(1-P^*)$
$[\gamma(c+d+\nabla),\gamma(c+d+\Delta)[$		See below (positive)	
$[\gamma(c+d+\Delta),+\infty[$	1	1	$(1-P^*)$

For the interval $[\gamma(c+d+\nabla),\gamma(c+d+\Delta)[$, it can be shown that

$$\frac{dA}{d\Delta}=\frac{dB}{d\Delta}=1+\frac{\gamma\nabla}{\Delta-\nabla}q^*>0.$$

Therefore,

$$\left[\frac{dA}{d\Delta}-\frac{dB}{d\Delta}P^*\right]=\left[1+\frac{\gamma\nabla}{\Delta-\nabla}q^*\right](1-P^*)>0.$$

Comparative statics with respect to ∇.

$(1-\gamma)\Gamma$	$\frac{dA}{d\nabla}$	$\frac{dB}{d\nabla}$	$\left[\frac{dA}{d\nabla}-\frac{dB}{d\nabla}P^*\right]$
$[0,\gamma c[$	0	$-(1-\gamma)$	$(1-\gamma)P^*$
$[\gamma c,\gamma(c+d+\nabla)[$	0	$-(1-\gamma)$	$(1-\gamma)P^*$
$[\gamma(c+d+\nabla),\gamma(c+d+\Delta)[$		See below (positive)	
$[\gamma(c+d+\Delta),+\infty[$	0	-1	P^*

For the interval $[\gamma(c+d+\nabla),\gamma(c+d+\Delta)[$, it can be shown that

$$\frac{dA}{d\nabla}=\gamma(1-q^*)\frac{\Delta}{\Delta-\nabla},\frac{dB}{d\nabla}=\gamma(1-q^*)\frac{\Delta}{\Delta-\nabla}-1.$$

Using these expressions, we have

$$\left[\frac{dA}{d\nabla}-\frac{dB}{d\nabla}P^*\right]=P^*+\gamma(1-q^*)\frac{\Delta}{\Delta-\nabla}(1-P^*)>0.$$

Comparative statics with respect to ρ.

The parameter ρ affects the optimal investment only through the parameters Δ and ∇, which have been shown to positively influence investment. Given the assumption $3t > \rho$, it can be readily shown that both Δ and ∇ increase with ρ, meaning that ρ has a positive impact on the investment level.

Comparative statics with respect to γ.

$(1-\gamma)\Gamma$	$\frac{dA}{d\gamma}$	$\frac{dB}{d\gamma}$	$\left[\frac{dA}{d\gamma}-\frac{dB}{d\gamma}P^*\right]$
$[0, \gamma c[$	$-(d+\Delta)$	$-(\nabla-\Delta)$	$-\Delta(1-P^*)-P^*\nabla-d$
$[\gamma c, \gamma(c+d+\nabla)[$	See below (negative)		
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	See below (positive)		
$[\gamma(c+d+\Delta), +\infty[$	0	0	0

For the interval $[\gamma c, \gamma(c+d+\nabla)[$, it can be easily shown that

$$\left[\frac{dA}{d\gamma}-\frac{dB}{d\gamma}P^*\right] = -\Delta(1-P^*)-P^*\nabla-d-c-\Gamma < 0.$$

Finally, for the interval $[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$, it can also be shown that

$$\left[\frac{dA}{d\gamma}-\frac{dB}{d\gamma}P^*\right] = \frac{\nabla(\Delta+c+d+\Gamma)}{\Delta-\nabla}(1-P^*) > 0.$$

Proof of Lemma 5: When neither firm commits, we face the situation analyzed in the previous sections. Let I_k^*, (for $k = i, j$) denote the optimal investment and let $P_k^* \equiv P(\theta_k, I_k^*)$ represent the resulting probability of success. In equilibrium, each firm earns the profit Π^*, as shown in Table A4 below. When both firms commit, we have a symmetrical situation, and firm i's equilibrium profits are given by

$$\Pi^C = \pi^* - c - (1 - P_i^C)d + \left(P_i^C(1 - P_j^C)\Delta - P_j^C(1 - P_i^C)\nabla\right) - I_i^C,$$

where $P_k^C \equiv P(\theta_k, I_k^C)$, for $k = i, j$, and where I_k^C matches the level of investment undertaken when the firm does not activate the device because $(1-\gamma)\Gamma \in [\gamma(c + d + \Delta), +\infty[$, so that it solves:

$$\left.\frac{\partial P_i}{\partial I_i}\right|_{I_i^C} \left[(d + \Delta) - (\Delta - \nabla)P_j^C\right] - 1 = 0.$$

Range for $(1-\gamma)\Gamma$ ↓	Profits Π^*
$[0, \gamma c[$	$\pi^* - (1-\gamma)(c + \Gamma) - (1 - P_i^*)(1-\gamma)d$ $+(1-\gamma)\left(P_i^*(1 - P_j^*)\Delta - P_j^*(1 - P_i^*)\nabla\right) - I_i^*$
$[\gamma c, \gamma(c + d + \nabla)[$	$\pi^* - c\left(P_i^* + (1-\gamma)(1 - P_i^*)\right) - (1 - P_i^*)(1-\gamma)(d + \Gamma)$ $+(1-\gamma)\left(P_i^*(1 - P_j^*)\Delta - P_j^*(1 - P_i^*)\nabla\right) - I_i^*$
$[\gamma(c + d + \nabla), \gamma\,(c + d + \Delta)[$	$\pi^* - c - (1 - P_i^*)d - (1 - P_i^*)(1 - P_j^*)(1 - q^*)\gamma\nabla$ $+\left(P_i^*(1 - P_j^*)\Delta - P_j^*(1 - P_i^*)\nabla\right) - I_i^*$
$[\gamma\,(c + d + \Delta), +\infty[$	$\pi^* - c - (1 - P_i^*)d + \left(P_i^*(1 - P_j^*)\Delta - P_j^*(1 - P_i^*)\nabla\right) - I_i^*$

Table B4: Equilibrium profits for all possible values of $(1-\gamma)\Gamma$.

If $\rho = 0$, the profits earned are given by: $\Pi^* = \pi^* - (1-\gamma)(c + d + \Gamma) + A'P(\theta, I^*) - I^*$ and $\Pi^C = \pi^* - c - d(1 - P(\theta, I^C)) - I^C$, where A' is given in Table 6. Using the envelope theorem, we have

$$\frac{d\Pi^*}{d\theta} = A'P_\theta|_{I=I^*} \text{ and } \frac{d\Pi^C}{d\theta} = dP_\theta|_{I=I^C}.$$

Given assumption (v) on the probability function, and since $A' \leq d$ and $I^* \leq I^C$, we can conclude that $0 < \frac{d\Pi^*}{d\theta} < \frac{d\Pi^C}{d\theta}$. Both functions are increasing, and Π^C increases at a higher rate. The difference between the two profits is given by:

$$\Pi^* - \Pi^C = \gamma(c + d) - (1-\gamma)\Gamma + A'P(\theta, I^*) - dP(\theta, I^C) + I^C - I^*.$$

At $\theta_i = \theta = 0$, by assumption, $P(0, I^*) = P(0, I^C) = 0$, we have $I^C = I^* = 0$, and

$$\Pi^* - \Pi^C = \gamma(c + d) - (1-\gamma)\Gamma \geq 0.$$

This is because we focus on the scenario of $(1-\gamma)\Gamma \leq \gamma\,(c + d)$, in which at least one of the firms may activate the device.

As $\theta \to +\infty$, the value of $\left(\frac{\theta}{1+\theta}\right)$ converges to 1. According to equation (9), in equilibrium, we have $\lim_{\theta \to +\infty} (1 + I^*)^2 = A'$ and $\lim_{\theta \to +\infty} (1 + I^C)^2 = d$.

We can re-write and simplify $(\Pi^{*}-\Pi^{C})$ as

$$\Pi^{*}-\Pi^{C}=\gamma(c+d)-(1-\gamma)\Gamma+(I^{*})^{2}-(I^{C})^{2}$$
$$=\gamma(c+d)-(1-\gamma)\Gamma+(1+I^{*})^{2}-(1+I^{C})^{2}-2(I^{*}-I^{C}).$$

Using the values of A' in Table 6, we find that:

- For any $(1-\gamma)\Gamma<\ \gamma c$, we have
$$\lim_{\theta\to+\infty}(\Pi^{*}-\Pi^{C})=\gamma c-(1-\gamma)\Gamma-2(I^{*}-I^{C})>0.$$
- For any $\gamma c<(1-\gamma)\Gamma<\ \gamma(c+d)$, we have
$$\lim_{\theta\to+\infty}(\Pi^{*}-\Pi^{C})=-2(I^{*}-I^{C})>0.$$

Therefore, $(\Pi^{*}-\Pi^{C})$ is always positive but the two profit functions become asymptotically close to each other as θ increases.

Analysis of the commitment game in the asymmetric cases: only one firm commits.

Without loss of generalities, we assume that only Firm 1 commits not to activate the device. Using Proposition 1, it is straightforward to show that Firm 2 relies on the following strategies at stage 3:

- If $(1-\gamma)\Gamma \in [0, \gamma c[$, Firm 2 always activates the device.
- If $(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d+\nabla)[$, Firm 2 activates the device when it fails to innovate.
- If $(1-\gamma)\Gamma \in [\gamma(c+d+\nabla), \gamma\,(c+d+\Delta)[$, Firm 2 activates the device when both firms fail to innovate.

Given Firm 2's strategy, we evaluate the equilibrium investments in each possible case, considering, without loss of generality, that Firm 1 commits not to use the device while Firm 2 does not.

- $\boldsymbol{(1-\gamma)\Gamma \in [0, \gamma c[}$

Firm 1 commits not to use a device, and its profits are given as follows:

$$\widehat{\Pi}^C = \pi^* - c - (1-P_1)d - \gamma(1-P_1)\nabla + (1-\gamma)(P_1(1-P_2)\Delta - P_2(1-P_1)\nabla) - I_1.$$

Firm 2 makes no such commitment, and its profits are given as follows:

$$\widehat{\Pi}^* = \pi^* - (1-\gamma)(c + (1-P_2)d + \Gamma) + \gamma(1-P_1)\Delta + (1-\gamma)(P_2(1-P_1)\Delta - P_1(1-P_2)\nabla) - I_2.$$

The optimal level of investments $\hat{I}_1^C$ and $\hat{I}_2^*$ are the solutions to

$$\left.\frac{\partial P_1}{\partial I_1}\right|_{\hat{I}_1^C}\left[(d + \Delta - \gamma(\Delta - \nabla)) - (1-\gamma)(\Delta - \nabla)P\big(\theta_2, \hat{I}_2^*\big)\right] - 1 = 0,$$

and

$$\left.\frac{\partial P_2}{\partial I_2}\right|_{\hat{I}_2^*}\left[(1-\gamma)(d+\Delta) - (1-\gamma)(\Delta - \nabla)P\big(\theta_1, \hat{I}_1^C\big)\right] - 1 = 0.$$

- $\boldsymbol{(1-\gamma)\Gamma \in [\gamma c, \gamma(c+d+\nabla)[}$

Firm 1 commits not to use a device, and its profits are given as follows:

$$\widehat{\Pi}^C = \pi^* - c - (1-P_1)d - \gamma(1-P_1)\nabla + (1-\gamma)(P_1(1-P_2)\Delta - P_2(1-P_1)\nabla) - I_1.$$

Firm 2 makes no such commitment, and its profits are given as follows:

$$\widehat{\Pi}^* = \pi^* - c\big(P_2 + (1-\gamma)(1-P_2)\big) - (1-P_2)(1-\gamma)(d+\Gamma) + \gamma(1-P_1)\Delta + (1-\gamma)(P_2(1-P_1)\Delta - P_1(1-P_2)\nabla) - I_2.$$

The optimal level of investments $\hat{I}_1^C$ and $\hat{I}_2^*$ are the solutions to

$$\left.\frac{\partial P_1}{\partial I_1}\right|_{\hat{I}_1^C}\left[(d + \Delta - \gamma(\Delta - \nabla)) - (1-\gamma)(\Delta - \nabla)P\big(\theta_2, \hat{I}_2^*\big)\right] - 1 = 0,$$

and

$$\left.\frac{\partial P_2}{\partial I_2}\right|_{\hat{I}_2^*}\left[(1-\gamma)(d + \Gamma + \Delta) - \gamma c - (1-\gamma)(\Delta - \nabla)P\big(\theta_1, \hat{I}_1^C\big)\right] - 1 = 0.$$

- $\boldsymbol{(1-\gamma)\Gamma \in [\gamma(c+d+\nabla), \gamma\,(c+d+\Delta)[}$

Firm 1 commits not to use a device, and its profits are given as follows:

$$\widehat{\Pi}^C = \pi^* - c - (1-P_1)d - \gamma(1-P_1)(1-P_2)\nabla + (P_1(1-P_2)\Delta - P_2(1-P_1)\nabla) - I_1.$$

Firm 2 makes no such commitment, and its profits are given as follows:

$$\begin{aligned}\widehat{\Pi}^* = \pi^* - c\big(1-\gamma(1-P_1)(1-P_2)\big) - (1-\gamma)(1-P_1)(1-P_2)\Gamma \\ - d(1-P_2)\big(P_1 + (1-\gamma)(1-P_1)\big) \\ +\gamma(1-P_1)(1-P_2)\Delta + (P_2(1-P_1)\Delta - P_1(1-P_2)\nabla) - I_2.\end{aligned}$$

The optimal level of investments $\hat{I}_1^C$ and $\hat{I}_2^*$ are the solutions to

$$\left.\frac{\partial P_1}{\partial I_1}\right|_{\hat{I}_1^C}\left[(d+\Delta+\gamma\nabla) - \big(\Delta - \nabla(1-\gamma)\big)P\big(\theta_2, \hat{I}_2^*\big)\right] - 1 = 0,$$

and

$$\left.\frac{\partial P_2}{\partial I_2}\right|_{\hat{I}_2^*}\Big[(1-\gamma)(d+\Gamma+\Delta) - \gamma c - P\big(\theta_1, \hat{I}_1^C\big)[(1-\gamma)(\Gamma+\Delta-\nabla) - \gamma(c+d+\nabla)]\Big] - 1 = 0.$$

From the above, we obtain that the investment decisions are once again inter-dependent, as the first-order condition can be written as

$$\frac{\partial P_i}{\partial I_i}\big[A_i - B_i P_j\big] - 1 = 0, \qquad \text{(A2)}$$

where the values of A_i $(i = 1,2)$ and B_i $(i = 1,2)$ are given in the tables below for Firm 1 and Firm 2.

$(1-\gamma)\Gamma \downarrow$	A_1	B_1
$[0, \gamma c[$	$(d+\Delta-\gamma(\Delta-\nabla))$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma c, \gamma(c+d+\nabla)[$	$(d+\Delta-\gamma(\Delta-\nabla))$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$(d+\Delta+\gamma\nabla)$	$\Delta-\nabla(1-\gamma)$

Table B5: Values of A_1 and B_1 for all possible values for $(1-\gamma)\Gamma$.

$(1-\gamma)\Gamma \downarrow$	A_2	B_2
$[0, \gamma c[$	$(1-\gamma)(d+\Delta)$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma c, \gamma(c+d+\nabla)[$	$(1-\gamma)(d+\Gamma+\Delta)-\gamma c$	$(1-\gamma)(\Delta-\nabla)$
$[\gamma(c+d+\nabla), \gamma(c+d+\Delta)[$	$(1-\gamma)(d+\Gamma+\Delta)-\gamma c$	$(1-\gamma)(\Gamma+\Delta-\nabla)$ $-\gamma(c+d+\nabla)$

Table A6: Values of A_2 and B_2 for all possible values for $(1-\gamma)\Gamma$.

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