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Upper Bounds on Risk Aversion under Mean-variance Utility

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Upper bounds on risk aversion under mean-variance utility

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Abstract:

Based on a simple prior, this note derives upper bounds for the coefficient of absolute & relative risk aversion if utility can be written as depending linearly on the mean and variance of income.

Keywords: risk aversion, mean-variance utility, risk tolerance

JEL Classification: D80

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[*] Email : kevin.denny@ucd.ie The basic result here I figured out c.1986 but I'm not inclined to rush matters. My thanks to Sarah Parlane for comments.

1. Introduction

Given expected utility maximization, the utility of an individual is a linear function of the mean and variance of their income under certain conditions: quadratic utility and/or a normal distribution of income. The former is a fairly restrictive assumption with some undesirable properties: utility is decreasing in the argument at some points and exhibits increasing absolute risk aversion. The latter assumption, normality of income, is probably a better rationale although it implies negative incomes having a non-zero probability.

Nonetheless the mean-variance framework is intuitive and is widely used, particularly in finance (for example Funga & Hsiehb 1999). If cognitive resources are limited, then an additional advantage to the mean-variance approach is that it is much simpler, requiring knowledge of only two parameters. D'Acremont & Bossaerts (2008) explore the behavioural foundations of the mean-variance approach and provide experimental evidence that it better characterizes agents' decisions in some circumstances i.e. as the number of possible states increase.

A feature of the model is that one can write an individual's certainty equivalent income as a linear function of the mean and variance. It does not appear to have been noticed that using a simple prior one can establish an upper bound to a standard measure of risk aversion. For the discrete case the certainty equivalent level of income is implicitly defined by:

$$U\left(\mu-\frac{1}{2}\gamma\sigma^2\right)=\sum_i \pi_i U(y_i) \qquad 1\geq\pi_{\mathrm{i}}\geq 0 \qquad \sum_i \pi_i = 1 \tag{1}$$

μ, σ^2 are the mean and variance respectively of income, $\gamma=-\frac{u''(y)}{u'(y)}$ is the coefficient of absolute risk aversion (CARA) and $\frac{1}{2}\gamma\sigma^2\equiv R$ is the risk premium, the amount individuals would be willing to pay to eliminate uncertainty. $-\frac{u''(y).y}{u'(y)}=\theta$ is the coefficient of relative risk aversion (CRRA)[1]. In the finance literature, the reciprocal of θ is known as the coefficient of relative risk tolerance (CRRT).

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[1] See J.Hirschliefer, J.G Riley (1992), section 2.3.1 for a discussion and derivation of the standard results. Nakamura (2015) considers more general nonlinear mean-variance utility models. Building on recent work in behavioural economics & psychology, O'Donoghue & Somerville (2018) consider modelling risk aversion in non-expected utility models.

2. Theory

Consider an individual who discards a fraction t of her income. While this reduces both the mean and variance of income, *a priori* one expects this to reduce certainty equivalent income. Given free disposability, in equilibrium an individual is no better off if she chooses to automatically discard a fraction of her income. One does not, in general, observe individuals disposing of income for the sole purpose of reducing the dispersion (although some people tithe a proportion of their income to religious organizations which might be considered a form of insurance).

From this it follows that:

$$\mu-\frac{1}{2}\gamma\sigma^2 \geq (1-t)\mu-\frac{1}{2}\gamma(1-t)^2\sigma^2 \tag{2}$$

$$\Rightarrow \frac{2\mu}{\sigma^2}\frac{1}{2-t} \geq \gamma \tag{3}$$

The limit of the LHS of (3) as t goes to 0 provides an upper bound to γ:

$$\frac{\mu}{\sigma^2} \geq \gamma \tag{4}$$

Hence:

$$\frac{2R}{\sigma^2}=\gamma \leq \frac{\mu}{\sigma^2} \Rightarrow \frac{R}{\mu} \leq \frac{1}{2} \tag{5}$$

(4) can be written in terms of the coefficient of relative risk aversion (CRRA), evaluated at the mean:

$$\left(\frac{\mu}{\sigma}\right)^2 \geq \theta(\mu) \tag{6}$$

Unlike (4), the LHS of (6) is unit free. So the results can be summarized as:

> **Lemma:** The coefficient of absolute risk aversion, under mean-variance utility, is weakly bounded from above by the ratio of the mean to the variance.

Corollary 1: The risk premium, as a proportion of the mean, is weakly bounded from above by 0.5.

Corollary 2: The coefficient of relative risk aversion, evaluated at the mean, is weakly bounded from above by the reciprocal of the squared coefficient of variation.

Alternatively, Corollary 2 could be re-stated as saying that the coefficient of relative risk tolerance, evaluated at the mean μ, is weakly bounded from below by the coefficient of variation squared.

Such bounds could be used to test the appropriateness of the mean-variance model or, if taking the latter as given, to evaluate estimates of risk aversion. How informative these bounds are will depend on the data.

3. Conclusion

The non-parametric approach used here has the advantage that it relies only on invoking a weak prior and data on the mean and variance of income[2]. A disadvantage is that the mean-variance model itself requires other assumptions, as discussed above, in addition to providing an upper-bound only. However other parametric methods of estimating risk preferences (e.g. Barsky *et al* 1997, Chetty 2006, Chiappori & Paiella 2011) require modelling labour supply responses, portfolio choices or hypothetical choices which depend on numerous other assumptions which may or may not be testable and are more demanding in terms of data.

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[2] The idea of using simple priors to establish bounds on parameters has been extensively developed by Charles Manski and others e.g. Manski (1990).

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