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Roberta Cardani, University of Pavia
Alessia Paccagnini, University College Dublin
Stelios Bekiros, European University Institute

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# The Effectiveness of Forward Guidance in an Estimated DSGE Model for the Euro Area: the Role of Expectations


Roberta Cardani*
University of Pavia

Alessia Paccagnini†
University College Dublin

Stelios Bekiros‡
European University Institute


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**Abstract**

We assess the effectiveness of the forward guidance undertaken by European Central Bank using a standard medium-scale DSGE model à la Smets and Wouters (2007). Exploiting data on expectations from surveys, we show that incorporating expectations should be crucial in performance evaluation of models for the forward guidance. We conduct an exhaustive empirical exercise to compare the pseudo out-of-sample predictive performance of the estimated DSGE model with a Bayesian VAR and a DSGE-VAR models. DSGE model with expectations outperforms others for inflation; while for output and short term-interest rate the DSGE-VAR with expectations reports the best prediction.



**Keywords:** *DSGE Bayesian estimation, Survey Professional Forecasts, Real Time data*

**JEL codes:** C52, C53, E58, E52



*Department of Economics and Management, University of Pavia, via San Felice, 5, 27100 Pavia, Italy. Email: roberta.cardani@unipv.it

†UCD School of Economics, Newman Building, Belfield, Dublin 4, Ireland. Email: alessia.paccagnini@ucd.ie.

‡European University Institute, Department of Economics, Via della Piazzuola; 43, I-50133, Florence, Italy; E-mail address: stelios.bekiros@eui.eu;

# 1 Introduction

In the aftermath of the financial crisis, the major economies have coped with the short term interest rate at the zero-lower bound (ZLB) or at a low constant level. In a such scenario, the ability of Central banks to stimulate the economy is weaken, since the policymaker looses its key instrument through which it provides macroeconomic stability in presence of shocks propagation, as discussed in Neri and Notarpietro (2014) and in Baurle and Kaufmann (2014).

Starting from 2013, the European Central Bank (ECB) has started to adopt the forward guidance as an extraordinary measure to enhancing the effectiveness of the monetary policy.[1] It grounds on the idea that monetary policy could have a larger effect on the longer-term interest rate if policymakers can commit themselves credibly to a path for future policy rate, as pointed out by Woodford (2012).[2]

The first theoretical contribution dates back to Krugman (1998), who analyzes the consequence deflationary slump in Japan during the 90s. Since then a growing theoretical framework has explored the consequence of forward guidance at zero-lower bound in a New Keynesian framework, such as in Eggertsson and Woodford (2003), Nakov (2008) and Fernández-Villaverde et al. (2012). However, only few studies have assessed quantitatively its effectiveness by means of VAR analysis. For example, Gertler and Karadi (2015) provide an empirical evidence through a proxy SVAR on forward guidance shocks in the US. They suggest that these shocks have larger effects on activity and inflation than standard contemporaneous shocks. Meanwhile, D'Amico and King (2015) perform a similar empirical analysis using sign-restrictions VAR framework.

Del Negro et al. (2015) show that, estimating a dynamic stochastic general equilibrium (DSGE) model, the forward guidance generates a very large response of aggregate variables, producing the so called "*forward guidance puzzle*". They explained this phenomenon by mean of the fact that only when agents discount future more heavily (as in the perpetual youth model) the announcements of policy changes in the future generate smaller effects on current aggregate variables. McKay et al. (2015) show that aggregate consumption in a model with heterogeneity and borrowing constraints does not suffer the same pitfall as in standard representative agent models, where the consumption response to current real rate cuts is just as large as that to interest rate cuts very far in the future.[3]

[1] On $4^{th}$ July 2013 the Governing Council of the ECB states its intention to keep interest rate at prevailing or lower levels "for an extended period of time", in order to lower the future rates below the market expectations. On 6 March 2014 the Governing Council reinforced the qualitative guidance formulation by spelling out more precisely the conditions for a low interest rate policy. However, as stressed by Filardo and Hofmann (2014) forward guidance has been used in a small number of inflation targeting Central banks during the 1990s, among which Japan, Norway, Iceland and Sweden.

[2] Note that there is no agreement in the profession on the beneficial effect of forward guidance. For example Kool, Middeldorp, and Rosenkranz (2011) show that under near-risk-neutrality of market participants, a crowding out of private information occurs, reducing forecast precision. Brzoza-Brzezina and Kot (2008) show that the benefits of publishing interest rate forecasts are marginal once macroeconomic forecasts are provided.

[3] Our analysis contains a caveat. Hirose and Inoue (2015) investigate quantitatively how and to what extent

We contribute the literature estimating under rational expectations a medium-scale DSGE model augmented by expectations to embody the role of the forward guidance. Our research questions are: *Does the DSGE model with expectations explain the behavior of the business cycle indicators in the Euro Area during the unconventional monetary policy period? Is a DSGE with the expectations a good instrument to model the forward guidance?*

To reply to our research questions, we compare the macroeconomic performance of a medium scale DSGE model à la Smets and Wouters (2007) with a medium scale DSGE model that includes Survey Professional Forecasts (SPF) as observables. If forward guidance successed, expectations will be disciplinated and the model would be able to forecast better the macrovariables. As far as we know, our paper is the first attempt to investigate the effectiveness of the forward guidance for the Euro Area in a DSGE model framework.

In a forecasting comparison, we show how the DSGE model with expectations outperforms the standard DSGE à la Smets and Wouters to forecast the inflation. Meanwhile, an hybrid model, as the DSGE-VAR à la Del Negro and Schorfheide (2004), augmented with expectations, reports the best predictions for the GDP growth rate and short term interest rate.

To understand the role of the expectations as tool to model the forward guidance, we implement a counterfactual exercise where we set the monetary policy rate at low constant level for several quarters. Estimating the standard Smets-Wouters model, we find what Del Negro et al. (2015) name as "forward guidance puzzle": an increase of the GDP growth rate and of the inflation. This experiment suggests us how the expectations modeled in a DSGE framework are a valid instrument to proxy the forward guidance.

Our main findings suggest how a DSGE model augmented with expectations helps the researcher to evaluate whether or not we can explain the key macroeconomic variables, as GDP growth rate, Inflation, and short term interest rate, during the unconventional monetary policy period.

The exploiting of observed expectations in a DSGE model is rarely used in the literature, with few exceptions. In this sense, our approach is akin to that adopted by Cole and Milani (2016), who exploit the SPF and real time data to estimate a DSGE model with friction to assess the empirical relationship between macroeconomic expectations and their realizations. They compare a DSGE-VAR à la Del Negro and Schorfheide (2004) to an unrestricted VAR and a DSGE model with cross-equation. They evidence that the DSGE model matches the data on expectation only by rejecting DSGE restrictions. On the other hand, Del Negro and Eusepi (2011) show that even if DSGE model fits well US macro data, it misspecifies in fitting the survey expectations.[4] Differently

parameter estimates can be biased in DSGE models lacking this constraint. They find that the bias becomes large as the probability of hitting the ZLB or the duration increase.

[4] The issue of expectations formation is beyond the scope of this paper. Along this framework see Omeno and Molnar (2015), Milani (2007, 2011), Slobodyan and Wouters (2012) and Granziera (2014), who show the important

from them, we address the forward guidance policy evaluation in Euro Area as a failure of rational expectations hypothesis to capture the behavior of expectations.

The remainder of the paper is organized as follows. Section 2 describes briefly the medium-scale DSGE model. Section 3 explains our empirical analysis. Section 4 evaluates the forecasting accuracy. Section 5 show the counterfactual experiment. Finally, Section 6 concludes. An appendix complements the paper by providing: Section A the sketch of the model while Section B the data description.

# 2 Model

Our model (SW, baseline) is based on Smets and Wouters (2007), which contains both nominal and real frictions affecting the choices of households and firms. Since the model is not the novelty of our paper, we briefly sketch its proprieties in the Appendix A.

The economy is composed of households, labour unions, labour packers, a productive sector and a monetary, and a fiscal authority. Households maximize their utility that depends on their level of consumption relative to an external habit component and leisure. Labour supplied by households is differentiated by a union with monopoly power setting sticky nominal wages à la Calvo. Households rent capital to firms and decide how much capital accumulation depending in the capital adjustment costs.

Nominal frictions affect the supply side and include both sticky intermediate goods and wages introduced by Calvo-pricing, with partial indexation for those firms who do not re-optimize their prices. In particular, intermediate firms decide how much capital they use depending on the capital utilization adjustment costs. Intermediate firms also decide how much differentiated labour they hire to produce differentiated goods and set their prices à la Calvo. Moreover, both wages and prices are partially indexed to lagged inflation when they are not re-optimized, introducing another source of nominal rigidity.

Monetary policy is set according to a Taylor rule and the government spending is exogenous.

The model contains 13 endogenous variables: output, consumption, investment, value of the capital stock, installed stock of capital, stock of capital, inflation, capital utilization rate, real rental rate on capital, real marginal cost, real wages, hours worked, and interest rate. Moreover, we consider 7 exogenous processes: total factor productivity, government spending, price and wage mark ups and monetary policy. All shocks are modelled as autoregressive processes with normal i.i.d. innovations, except for the price and wage mark-ups which are assumed to follow a first order autoregressive moving average process. Note that the model is detrended with respect to the deterministic growth rate of the labor- augmenting technological progress and linearized around

role of expectations in explaining the inflation process by means of different learning assumptions.

the steady state of the detrended variables.

# 3 Empirical Analysis

To investigate empirically the effectiveness of the forward guidance we focus on Euro Area data quarterly time series from 1999 to 2015 exploding real time data.

Our empirical analysis is composed of three steps.

As first step, we compare posteriors of the estimated parameters of SW model with the same model and same database but augmented with expectations (SW with expectations) as observables incorporating the Survey Professional Forecast (SPF). The estimation procedure is implemented using Bayesian techniques as described in Smets and Wouters (2007).

As second step, in a pseudo out-of-sample forecasting exercise, we assess the prediction ability of the two DSGE models (baseline and with expectations) from 2012:Q4 to 2015:Q4. In our horserace, we include a Bayesian VAR model with priors à la Sims and Zha (1998) and the hybrid DSGE-VAR introduced by Del Negro and Schorfheide (2004).

As third step, we design the counterfactual experiment to understand whether the expectations are a valid and good instrument to proxy the forward guidance in a DSGE model framework. As the Euro Area reached the lower bound in 2012:Q4, we assume a monetary policy rate equals to 20 basis point[5] for the period from 2012:Q4 to 2013:Q3. Thus, we compare the results with forecasting performance of the SW baseline, in which we don't take into account the expectations as observables.

## 3.1 Data

We estimate the SW model on Euro Area data. We use quarterly (first vintage) real time data on real GDP (growth rate) and HICP inflation, collected from Eurostat starting from 1999:Q2.

Moreover, we consider the short term nominal interest rate (Euribor), employment, private consumption (growth rate), investments (growth rate) and compensation per employee from ECB Data Warehouse, as observable variables to be matched in the estimation. Output, consumption, investments, and wages are transformed in log differences; instead, total employment has been detrended with a HP trend.

Our survey measures are expectations for the one-year ahead inflation rate and real GDP growth from the Survey of Professional Forecasts (SPF). This survey is collected at quarterly series from

[5] This lower limit for the Euro Area interest rate is the average over the period 2012:Q4 to 2014:Q4, with very small variations. Thus we assume this value in the exercise below.

1999:Q1. So our data sample cover the period from 1999:Q2 to 2015:Q4.[6]

Figure 1 shows the different path between the median SPF inflation and real-time data. In particular, inflation realization appears to be quite different from SPF.

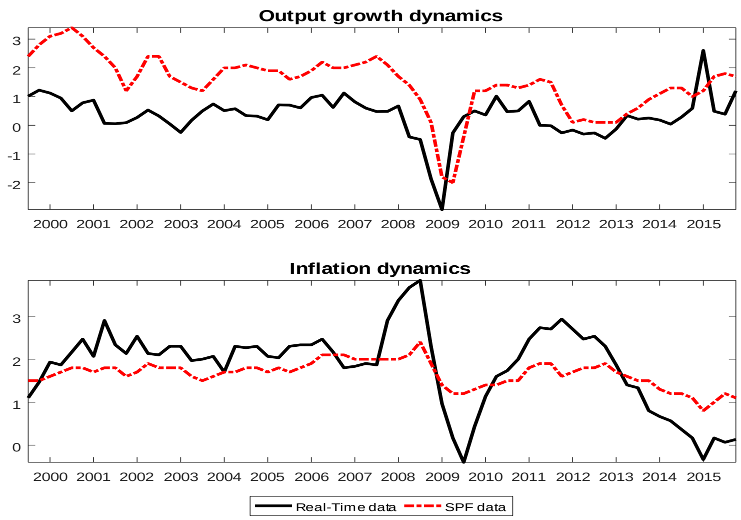


Figure 1: Realization and SPF series

## 3.2 Estimation details

The DSGE model is estimated using Bayesian estimation technique. Employing the random walk Metropolis-Hasting algorithm, we run four chains of 250.000 draws of all the possible realizations $\xi$ for each parameter in order to obtain its posterior distribution.[7]

The measurement equations, which relate the model to the observed variables, in case of the SW baseline are as follows:

$$Y_t = \begin{bmatrix} \Delta \ln y_t \\ \Delta \ln c_t \\ \Delta \ln i_t \\ \Delta \ln w_t \\ \ln e_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{e} \\ \overline{\pi}_* \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ e_t \\ \pi_t \\ r_t \end{bmatrix}, \tag{1}$$

where ln denotes 100 times log and $\Delta \ln$ refers to the log difference. $\overline{\gamma} = 100(\gamma - 1) + \gamma_t$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages, where $\gamma_t$

[6]We start the sample with the first realise of the Survey Professional Forecaster. A detailed description of the dataset used for estimation can be found in the Appendix B.

[7]The DSGE models are estimated using Dynare toolbox for Matlab.

is the permanent technology shock. Further, $\overline{\pi}_* = 100(\overline{\pi}-1)$ is the quarterly steady-state inflation rate, $\overline{r} = 100(\beta^{-1}\gamma\overline{\pi} - 1)$ is the steady-state nominal interest rate, and $\overline{e}$ is the steady-state employment, which is normalized at zero.[8]

In case of the SW estimated with SPF, the measurement equations set (1) includes two additional equations for expectations as follows:

$$\begin{bmatrix} E_t\Delta y_{t+1}^{obs} \\ E_t\pi_{t+1}^{obs} \end{bmatrix} = \begin{bmatrix} E_t y_{t+1} \\ E_t\pi_{t+1} \end{bmatrix} + \begin{bmatrix} \hat{\varepsilon}_t^{y+1} \\ \hat{\varepsilon}_t^{\pi+1} \end{bmatrix},$$

where $E_t\Delta y_{t+1}^{obs}$ and $E_t\pi_{t+1}^{obs}$ denote respectively one-period-ahead real GDP growth expectations and inflation expectations. We interpret the survey data as a noisily measure of model consistent with rational expectations, following Cole and Milani (2016).

As argued by Ormeno and Molnar (2015), when survey data is used as an observable in the estimation, agent's expectations on inflation and output have to explain not only the model equations but also the SPF survey.

### 3.3 Calibration and priors

The model is calibrated as described in Table 1. In particular, we set the discount factor $\beta$ at the standard level 0.99, in line with a steady-state real interest rate of about 4%. The depreciation rate $\delta$ is 0.025 per quarter (approx. 10% per year). The Kimball aggregators in the goods and labor market are equal to 10, and the steady state gross wage and price mark-up is set respectively to 1.61 and 1.5.

The share of government spending to GDP ratio corresponds to the average share in the period (1999-2015) and is fixed at 20%, corresponding to the average share in this period for Euro Area.

Table 1: Calibrated parameters

| parameter | | value |
|---|---|---|
| $\sigma_c$ | Intertemporal elasticity of substitution | 1 |
| $\beta$ | discount factor | 0.99 |
| $\delta$ | capital depreciation rate | 0.025 |
| $\eta_p$ | Kimball aggregator in the goods markets | 10 |
| $\eta_w$ | Kimball aggregator in the labour markets | 10 |
| $\lambda_p$ | Gross steady state price markup | 1.61 |
| $\lambda_w$ | Gross steady state wage markup | 1.5 |
| $\frac{G}{Y}$ | Government share of output | 0.19 |

[8] Following CCW, we relate the employment variable, $e_t$, to the unobserved worked-hours variable, $h_t$, by means of $\hat{e}_t = \frac{\beta}{1+\beta}E_t\hat{e}_{t+1} + \frac{1}{1+\beta}\hat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e}\left(\hat{h}_t - \hat{e}_t\right)$, $\xi_e$ determines the sensitivity of employment with respect to worked hours.

To make our economy as representative as possible, priors for parameters and shocks are set according to the literature on Euro area, as in Smets and Wouters (2005) and Coenen et. al. (2012). Values are reported in table 2:

Table 2: Priors

| | Parameters | shape |
|---|---|---|
| $\phi_l$ | inverse of Frish elasticity | N(2,0.75) |
| $b$ | habits in consumption | B(0.7,0.1) |
| $\bar{\pi}_*$ | steady state inflation | G(0.62,0.1) |
| $\gamma$ | SS output growth | N(0.5,0.05) |
| $\varphi$ | investment adjustement cost | N(4,1.5) |
| $\alpha$ | capital share | N(0.3,0.05) |
| $\xi_p$ | price rigidity | B(0.75,0.1) |
| $\iota_p$ | price indexation to past inflation | B(0.75,0.1) |
| $\xi_w$ | wage rigidity | B(0.75,0.1) |
| $\iota_w$ | wage indexation to past inflation | B(0.75,0.1) |
| $\xi_e$ | Calvo employment | B(0.5,0.15) |
| $\Phi_r$ | interest rate smoothing | B(0.75,0.1) |
| $\Phi_\pi$ | Taylor rule parameter on inflation | N(1.5,0.25) |
| $\Phi_y$ | Taylor rule parameter on output | N(0.12,0.05) |
| $\Phi_{\Delta y}$ | Taylor rule parameter on change in output | N(0.12,0.05) |
| | Shocks | |
| $\rho_a, \rho_b, \rho_i, \rho_w, \rho_r, \rho_p, \rho_g, \rho_\pi^e, \rho_y^e$ | AR coefficient of shocks | B(0.5,0.15) |
| $\mu_w, \mu_p$ | MA coefficient of shocks | B(0.5,0.15) |
| $\sigma_a, \sigma_b, \sigma_i, \sigma_w, \sigma_r, \sigma_p, \sigma_g, \sigma_\pi^e, \sigma_y^e$ | standard deviation shocks | IG(0.1,$\infty$) |

## 3.4 Alternative Estimation models

We compare the prediction ability of the two DSGE models considering a Bayesian VAR (BVAR) model and the DSGE-VAR à la Del Negro and Schorfheide (2004).

### 3.4.1 Bayesian VAR

As discussed by Smets and Wouters (2007), the BVAR à la Sims and Zha (1998) is a good alternative model to forecast macroeconomic series such as GDP growth rate, CPI, and interest rate. The main advantage of considering this kind of BVAR is that it combines a Minnesota-type prior as in Litterman (1981, 1986) with a unit-root prior which considers the degree of persistence and cointegration in the variables. In the comparison analysis, we estimate the BVAR à la Sims and Zha (1998) with two lags as suggested by Akaike and Schwartz Information Criteria.

### 3.4.2 DSGE-VAR à la Del Negro and Schorfheide (2004)

Based on the study of Ingram and Whiteman (1994), Del Negro and Schorfheide (2004) designed the DSGE-VAR approach to improve forecasting and monetary policy analysis with VARs. Their approach is to use the DSGE model to build prior distributions for the VAR. Basically, the estimation initializes with an unrestricted VAR of order p:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + ... + \Phi_p Y_{t-p} + u_t. \tag{2}$$

In compact format:

$$Y = X\Phi + U, \tag{3}$$

where $Y$ is a $(T \times n)$ matrix with rows $Y_t'$, $X$ is a $(T \times k)$ matrix ($k = 1 + np, p =$number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, $U$ is a $(T \times n)$ matrix with rows $u_t'$ and $\Phi$ is a $(k \times n) = [\Phi_0, \Phi_1, ..., \Phi_p]'$.The one-step-ahead forecast errors $u_t$ have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of $Y$. The log-likelihood function of the data is a function of $\Phi$ and $\Sigma_u$ :

$$L(Y|\Phi, \Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2} tr\left[\Sigma_u^{-1}\left(\mathbf{Y'Y} - \mathbf{\Phi'}X'Y - Y'X\Phi + \Phi'\mathbf{X'X\Phi}\right)\right]\right\}. \tag{4}$$

The prior distribution for the VAR parameters proposed by Del Negro and Schorfheide (2004) is based on the statistical representation of the DSGE model given by a VAR approximation. Let $\Gamma_{xx}^*$, $\Gamma_{yy}^*$, $\Gamma_{xy}^*$ and $\Gamma_{yx}^*$ be the theoretical second-order moments of the variables $Y$ and $X$ implied by the DSGE model, where:

$$\begin{gathered} \Phi^*(\theta) = \Gamma_{xx}^{*-1}(\theta)\,\Gamma_{xy}^*(\theta)\,, \\ \Sigma^*(\theta) = \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\,\Gamma_{xx}^{*-1}(\theta)\,\Gamma_{xy}^*(\theta)\,. \end{gathered} \tag{5}$$

The moments are the dummy observation priors used in the mixture model. These vectors can be interpreted as the probability limits of the coefficients in a VAR estimated on the artificial observations generated by the DSGE model. Conditional on the vector of structural parameters in the DSGE model $\theta$, the prior distributions for the VAR parameters $p(\Phi, \Sigma_u|\theta)$ are of the Inverted-Wishart (IW) and Normal forms:

$$\begin{gathered} \Sigma_u\,|\theta \sim IW\left((\lambda T \Sigma_u^*(\theta)\,, \lambda T - k, n\right), \\ \Phi\,|\Sigma_u, \theta \sim N\left(\mathbf{\Phi}^*(\theta)\,, \Sigma_u \otimes (\lambda T \Gamma_{XX}(\theta))^{-1}\right), \end{gathered} \tag{6}$$

where the parameter $\lambda$ controls the degree of model misspecification with respect to the VAR: for small values of $\lambda$ the discrepancy between the VAR and the DSGE-VAR is large and a sizeable distance is generated between the unrestricted VAR and DSGE estimators. Large values of $\lambda$ correspond to small model misspecification and for $\lambda = \infty$ beliefs about DSGE misspecification degenerate to a point mass at zero. Bayesian estimation could be interpreted as estimation based on a sample in which data are augmented by a hypothetical sample in which observations are generated by the DSGE model, the so-called dummy prior observations (Theil and Goldberg, 1961; Ingram and Whiteman, 1994). Within this framework $\lambda$ determines the length of the hypothetical sample. The posterior distributions of the VAR parameters are also of the Inverted-Wishart and Normal forms. Given the prior distribution, posterior distributions are derived by the Bayes theorem:

$$\Sigma_u \left| \theta, \mathbf{Y} \right. \sim IW\left( \left( \lambda+1 \right) T \hat{\Sigma}_{u,b}\left( \theta \right), \left( \lambda+1 \right) T - k, n \right), \tag{7}$$

$$\Phi \left| \Sigma_u, \theta, \mathbf{Y} \right. \sim N\left( \hat{\mathbf{\Phi}}_b\left( \theta \right), \Sigma_u \otimes \left[ \lambda T \Gamma_{XX}\left( \theta \right) + \mathbf{X}'\mathbf{X} \right]^{-1} \right), \tag{8}$$

$$\hat{\Phi}_b\left( \theta \right) = \left( \lambda T \Gamma_{XX}\left( \theta \right) + \mathbf{X}'\mathbf{X} \right)^{-1} \left( \lambda T \Gamma_{XY}\left( \theta \right) + \mathbf{X}'\mathbf{Y} \right), \tag{9}$$

$$\hat{\Sigma}_{u,b}\left( \theta \right) = \frac{1}{\left( \lambda+1 \right) T} \left[ \left( \lambda T \Gamma_{YY}\left( \theta \right) + \mathbf{Y}'\mathbf{Y} \right) - \left( \lambda T \Gamma_{XY}\left( \theta \right) + \mathbf{X}'\mathbf{Y} \right) \hat{\mathbf{\Phi}}_b\left( \theta \right) \right], \tag{10}$$

where the matrices $\hat{\Phi}_b\left( \theta \right)$ and $\hat{\Sigma}_{u,b}\left( \theta \right)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE. Equations (7) and (8) show that the smaller $\lambda$ is, the closer the estimates are to the OLS estimates of an unrestricted VAR. Instead, the higher $\lambda$ is, the closer the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\hat{\Phi}_b\left( \theta \right)$ and $\hat{\Sigma}_{u,b}\left( \theta \right)$). In order to obtain a non-degenerate prior density (6), which is a necessary condition for the existence of a well-defined Inverse-Wishart distribution and for computing meaningful marginal likelihoods, $\lambda$ has to be greater than $\lambda_{MIN}$ :

$$\begin{aligned} \lambda_{MIN} &\geq \frac{n+k}{T}; k = 1 + p \times n \\ p &= \text{lags} \\ n &= \text{endogenous variables.} \end{aligned}$$

Hence, the optimal lambda must be greater than or equal to the minimum lambda $\left( \hat{\lambda} \geq \lambda_{MIN} \right)$. Essentially, the DSGE-VAR tool allows the econometrician to draw posterior inferences about the DSGE model parameters $\theta$. Del Negro and Schorfheide (2004) explain that the posterior estimate of $\theta$ has the interpretation of a minimum-distance estimator, where the discrepancy between the OLS estimates of the unrestricted VAR parameters and the VAR representation of the DSGE model is a sort of distance function. The estimated posterior of parameter vector $\theta$ depends on the hyperparameter $\lambda$. When $\lambda \to 0$, in the posterior of the parameters are not informative, so the DSGE model is of no use in explaining the data. Unfortunately, the posteriors (8) and (7) do not have a closed form and we need a numerical method to solve the problem. The posterior simulator used by Del Negro and Schorfheide (2004) is the Markov Chain Monte Carlo Method and the algorithm used is the Metropolis-Hastings acceptance method. This procedure generates a Markov Chain from the posterior distribution of $\theta$ and this Markov Chain is used for Monte Carlo simulations. The optimal $\lambda$ is given by maximizing the log of the marginal data density

$$\hat{\lambda} = \underset{\lambda \geqslant \lambda_{MIN}}{\arg\max} \ln p(\mathbf{Y}|\lambda)$$

According to the optimal lambda, $\hat{\lambda}$, a corresponding optimal mixture model is chosen. This hybrid model is called DSGE-VAR and $\hat{\lambda}$ is the weight of the priors. It can also be interpreted as the restriction of the theoretical model on the actual data. We estimate the DSGE-VAR with two lags as suggested by Akaike and Schwartz Information Criteria.

# 4 Results

Table 3 and 4 summarize estimation results for the SW model and the SW model augmented by expectations, reporting, the mean and the 5 and 95 percentiles of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm.

The posterior distributions of the SW baseline model for most of the parameters do not differ significantly from the literature. The steady state growth rate is estimated to be around 0.5, which is somewhat greater than the average growth rate of output over the sample. The posterior mean of the steady state inflation rate over the full sample is about 3.5% on an annual basis. The mean of the discount rate is estimated to be quite small (0.7% on an annual basis).

The productivity, the government spending, and the wage mark-up processes are estimated to be the most persistent with an AR(1) coefficient of 0.99, 0.84 and 0.71, respectively. The mean of the standard error of the shock to the productivity process is relatively high, meaning that at long horizons most of the forecast error variance of the real variables will be explained by this shock. In contrast, both the persistence and the standard deviation of inertia in monetary policy is relatively low (0.09). Policy reacts strongly to inflation expectations (1.52), but does not respond to output gap (0.049) and to change in the output-gap (0.18) in the short run.

Most source of endogenous persistence lose some of their importance when expectations are taken into account. The estimates of habit formation parameter, $b$, and the investment adjustment costs, $\varphi$, become smaller with compare to the Smets and Wouters's economy (respectively from

Table 3: Posteriors distributions of parameters

| | | Priors | SW model+Expectations | | SW model | |
|---|---|---|---|---|---|---|
| Parameters | | | Mean | 90% interval | Mean | 90% interval |
| $\phi_l$ | inverse of Frish elasticity | N(2; 0.75) | 1.290 | (0.221; 2.224) | 1.562 | (0.721; 2.347) |
| $b$ | habits in consumption | B(0.7; 0.1) | 0.734 | (0.671; 0.798) | 0.827 | (0.785; 0.871) |
| $\bar{\pi}_*$ | steady state inflation | G(0.62; 0.1) | 0.902 | (0.755; 1.070) | 0.508 | (0.351; 0.662) |
| $\gamma$ | SS output growth | N(0.5; 0.05) | 0.412 | (0.361; 0.455) | 0.546 | (0.476; 0.616) |
| $\varphi$ | investment adjustment cost | N(4; 1.5) | 5.567 | (3.728; 7.942) | 6.432 | (4.881; 7.972) |
| $\sigma_u$ | Capital utilization | B(0.5; 0.15) | 0.780 | (0.668; 0.901) | 0.704 | (0.562; 0.852) |
| $\alpha$ | capital share | N(0.3; 0.05) | 0.271 | (0.234; 0.308) | 0.259 | (0.203; 0.313) |
| $\phi_p$ | Fixed cost in production | N(1.25; 0.125) | 1.830 | (1.648; 2.000) | 1.653 | (1.512; 1.791) |
| $\xi_p$ | price rigidity | B(0.75; 0.1) | 0.927 | (0.893; 0.953) | 0.525 | (0;437; 0.612) |
| $\iota_p$ | price indexation | B(0.75; 0.1) | 0.336 | (0.300; 0.396) | 0.745 | (0.599; 0.888) |
| $\xi_w$ | wage rigidity | B(0.75; 0.1) | 0.817 | (0.753; 0.886) | 0.741 | (0.604; 0.890) |
| $\iota_w$ | wage indexation | B(0.75; 0.1) | 0.394 | (0.235; 0.569) | 0.681 | (0.504; 0.869) |
| $\xi_e$ | Calvo employment | B(0.5; 0.15) | 0.752 | (0.705; 0.803) | 0.815 | (0.783; 0.848) |
| $\Phi_r$ | interest rate inertia | B(0.75; 0.1) | 0.979 | (0.968; 0.90) | 0.965 | (0.949; 0.981) |
| $\Phi_\pi$ | Taylor rule inflation | N(1.5; 0.25) | 1.597 | (1.322; 1.959) | 1.525 | (1.176; 1.852) |
| $\Phi_y$ | Taylor rule on GDP | N(0.12; 0.05) | 0.161 | (0.105; 0.229) | 0.049 | (-0.036; 0.160) |
| $\Phi_{\Delta y}$ | Taylor rule change GDP | N(0.12; 0.05) | 0.203 | (0.157; 0.246) | 0.184 | (0.149; 0.220) |
| LDD | | | -415 | | -299 | |

0.8 to 0.7 and 6.4 to 5.5). Similarly, the price and wage indexation as well as price rigidity are now much smaller.

Instead, the persistences of the interest rate inertia, government spending and preference are increasing. Overall, we observe that incorporating expectations implies a lower estimates of the parameters characterizing both endogenous and exogenous sources of persistence than the baseline model. Consequently, the expectation shocks play a crucial role in the estimation procedure.

These results are in line with the bounded rationality findings, such as Eusepi and Preston (2011).

Table 4: Posteriors distributions of shocks

| | | Priors | SW model+Expectations | | SW model | |
|---|---|---|---|---|---|---|
| Shocks | | | Mean | 90% interval | Mean | 90% interval |
| $\rho_a$ | AR coeff. productity | B(0.5; 0.15) | 0.980 | (0.971; 0.990) | 0.992 | (0.988; 0.996) |
| $\rho_b$ | AR coeff. preference | B(0.5; 0.15) | 0.911 | (0.870; 0.955) | 0.584 | (0.366; 0.930) |
| $\rho_p$ | AR coeff. price markup | B(0.5; 0.15) | 0.211 | (0.026; 0.634) | 0.603 | (0.422; 0.790) |
| $\rho_w$ | AR coeff. wage markup | B(0.5; 0.15) | 0.556 | (0.414; 0.669) | 0.718 | (0.502; 0.921) |
| $\rho_g$ | AR coeff. governmet spending | B(0.5; 0.15) | 0.888 | (0.827; 0.949) | 0.849 | (0.781; 0.919) |
| $\rho_i$ | AR coeff. investment-specific | B(0.5; 0.15) | 0.392 | (0.229; 0.531) | 0.388 | (0.222; 0.552) |
| $\rho_r$ | AR coeff. monetary | B(0.5; 0.15) | 0.310 | (0.142; 0.475) | 0.324 | (0.167; 0.475) |
| $\rho^e_{pic}$ | AR coeff. inflation expectations | B(0.5; 0.15) | 0.722 | (0.543; 0.979) | - | - |
| $\rho^e_y$ | AR coeff. GDP expectations | B(0.5; 0.15) | 0.987 | (0.979; 0.995) | - | - |
| $\mu_p$ | MA coeff. price markup | B(0.5; 0.15) | 0.384 | (0.267; 0.654) | 0.428 | (0.236; 0.620) |
| $\mu_w$ | MA coeff. wage markup | B(0.5; 0.15) | 0.852 | (0.801; 0.915) | 0.470 | (0.280; 0.666) |
| $\sigma_a$ | St. Dev coeff. productity | IG(0.4; $\infty$) | 0.5555 | (0.430; 0.685) | 0.726 | (0.553; 0.892) |
| $\sigma_b$ | St. Dev preference | IG(0.4; $\infty$) | 0.3498 | (0.220;0.469) | 0.345 | (0.130; 0.559) |
| $\sigma_p$ | St. Dev price markup | IG(0.4; $\infty$) | 0.2729 | (0.228;0.315) | 0.224 | (0.168; 0.278) |
| $\sigma_w$ | St. Dev wage markup | IG(0.4; $\infty$) | 0.3613 | (0.278;0.440) | 0.476 | (0.364; 0.593) |
| $\sigma_g$ | St. Dev governmet spending | IG(0.4; $\infty$) | 0.0916 | (0.076;0.107) | 0.538 | (0.458; 0.617) |
| $\sigma_i$ | St. Dev investment-specific | IG(0.4; $\infty$) | 0.635 | (0.533;0,761) | 0.332 | (0.262; 0.400) |
| $\sigma_r$ | St. Dev monetary inertia | IG(0.4; $\infty$) | 0.7261 | (0.606;0.841) | 0.096 | (0.076; 0.116) |
| $\sigma^e_{pic}$ | St. Dev inflation expectations | IG(0.4; $\infty$) | 0.0704 | (0.052;0.135) | - | - |
| $\sigma^e_y$ | St. Dev GDP expectations | IG(0.4; $\infty$) | 0.7401 | (0.787;0.877) | - | - |

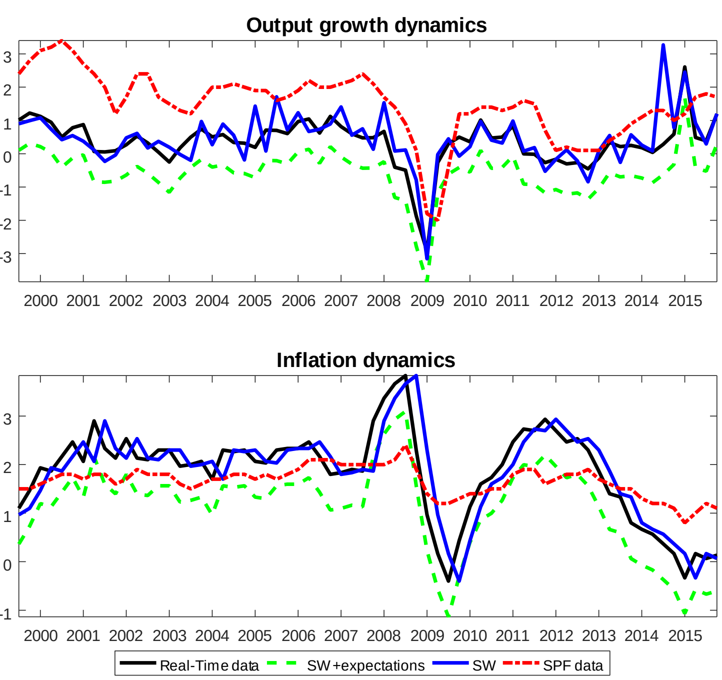


Figure 2: Survey data and model-implied series

Figure 2 outlines the relation between the median SPF inflation and output gap, the observations and the realization of our models. The paths generated by both models are quite different from the SPF data and more similar to real time data, especially in the baseline model for the ouput. However, the SW model augmeted by expectations seems to fit better on average inflation dynamics.

# 5 Forecasting evaluation of the role of expectations

We perform a forecasting comparison among the two DSGE models, a BVAR, and DSGE-VAR models as described in Table 6. All these models are estimated from 1999:Q2 to 2012:Q3 and the pseudo out-of-sample is set from 2012:Q4 to 2015:5.

For the SW baseline and the SW with expectations, we generate unconditional forecasts taking each 20th draw from the final 250,000 parameter draws (with the first 50,000 draws used as burn-in period) produced by the Metropolis-Hastings algorithm, which gives us 10,000 draws from the posterior distribution. The point forecasts are calculated as means of these draws. For more technical details, see Kolasa et al. (2012) and Kolasa and Rubaszek (2014).

Table 5: Model implied forecasting

| | 2012q4 | 2013q4 | 2014q4 | 2015q4 |
|---|---|---|---|---|
| *GDP growth* | | | | |
| Observed | -0.45 | 0.25 | 0.59 | 1.2 |
| SW model | -0.31 | 0.69 | 1.26 | 1.26 |
| SW + expectations | -0.35 | 0.63 | 0.98 | 1.21 |
| BVAR | -0.08 | 0.25 | 0.28 | 0.17 |
| DSGEVAR | -0.31 | 0.79 | 1.08 | 0.97 |
| DSGEVAR + expectations | -0.41 | 0.35 | 0.47 | 1.16 |
| *Inflation* | | | | |
| Observed | 2.3 | 0.8 | 0.17 | 0.13 |
| SW model | 2.47 | 1.32 | 0.71 | 0.51 |
| SW + expectations | 2.31 | 0.8 | 0.3 | 0.47 |
| BVAR | 2.43 | 2.49 | 2.72 | 2.87 |
| DSGEVAR | 2.44 | 2.02 | 1.38 | 0.92 |
| DSGEVAR + expectations | 2.31 | 0.92 | 0.95 | 0.88 |
| *Interest rate* | | | | |
| Observed | 0.15 | 0.13 | 0.08 | 0.02 |
| SW model | 0.32 | 0.22 | 0.42 | 0.81 |
| SW + expectations | 0.22 | 0.21 | 0.35 | 0.59 |
| BVAR | 0.17 | 0.10 | 0.09 | 0.06 |
| DSGEVAR | 0.19 | 0.09 | 0.13 | 0.10 |
| DSGEVAR + expectations | 0.22 | 0.08 | 0.10 | 0.08 |

Table 6 reports the observed values and the forecasted ones for three macroeconomic variables, GDP growth, Inflation, and interest rate, considering the periods: 2012:Q4, 2013:Q4, 2014:Q4, and 2015:Q4.

The forecasted values are reported for the SW baseline, the SW with expectations, the Bayesian VAR (BVAR), and the DSGE-VAR for both DGSE models.

At the first glance, focusing on the DSGE model forecasts, we note that the SW with expectations outperforms the SW baseline for all three key macroeconomic variables. In particular, for the inflation, the forecasted values are very close to the observed values especially in the short horizons, 2012:Q4 and 2013:Q4. Figure 3 shows the difference among the observed values and two forecasted ones. Graphically, we note that the DSGE models are weak to predict the interest rate at the long horizons. Several papers point out that the Bayesian VAR is the most suitable model to predict business cycle indicators, such as the GDP growth rate and the interest rate

We estimate a BVAR à la Sims and Zha (1998) to provide a comparison with an alternative model.

As discussed for the Euro Area in Bekiros and Paccagnini (2016), the BVAR outperforms the DSGE models for the GDP growth rate and for the short term interest rates. For these two macroeconomic variables, it seems that the DSGE models fail to predict.

For this reason, we introduce in our forecasting comparison the DSGE-VAR à la Del Negro and Schorfheide (2004). The DSGE-VAR is an hybrid model combining information from the observed time series and from the theoretical DSGE model[9]. We estimate the DSGE-VAR for the SW baseline and the SW with expectations. In both cases, the hyperparameter, $\lambda$, which indicates whether the posterior of the parameters are informative, is close to 1. Hence, we can conclude that the DSGE is far to be misspecified[10].

The DSGE-VAR with expectations outperforms the DSGE-VAR without expectations. In particular, the DSGE-VAR with expectations report similar forecasts as ones produced by the BVAR for the GDP growth and the interest rate.

As main findings, we can conclude that the inflation is well predicted by the SW with expectations, while the GDP growth and the interest rate are better predicted by the BVAR and the DSGE-VAR with expectations.

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[9] As discussed in Sims (2007), the DSGE-VAR is a "Bayesian VAR" with the priors derived from a theoretical DSGE model. DSGE-VAR combines the advantage of the VAR model class to forecast with priors with model information. As pointed out by Sims (2007), DSGE-VAR does this "by modeling the data as a VAR — that is, without the tight parametric restrictions implied by a DSGE — but using a DSGE, and prior beliefs about the parameters of the DSGE, to generate a prior distribution for the parameters of the VAR".

[10] For a detailed discussion about the role of the $\lambda$ hyperparameter, see Paccagnini (2010 and 2011), Bekiros and Paccagnini (2014) among other.

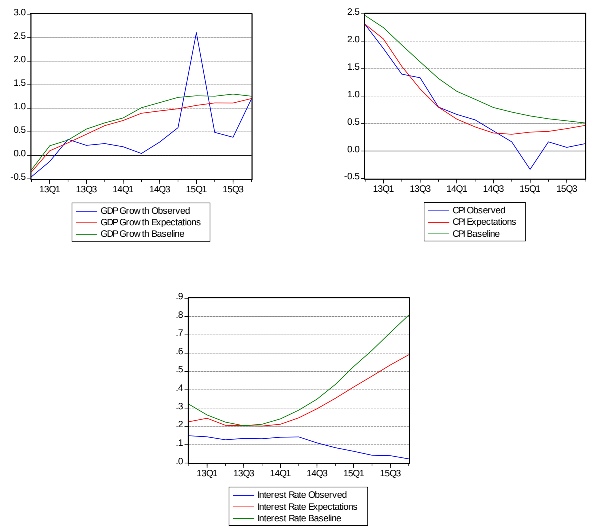


Figure 3: Forecasting comparison

# 6 Counterfactual Experiment

We design a counterfactual experiment to investigate whether the DSGE model augmented with expectations is a valid and good instrument to proxy the forward guidance in a DSGE model framework. We set our exercise akin to a counterfactual experiment proposed in Del Negro et al. (2015). We assume a monetary policy rate equals to 0.20 percent for the period from 2012:Q4 to 2013:Q3. We repeat the forecasting exercise estimating only the SW baseline.

The purpose of this analysis is to evidence if an alternative approach to the SW with expectations is a suitable way to proxy the forward guidance.

Table 6: Counterfactual Experiment

| | 2013q4 | 2014q4 | 2015q4 |
|---|---|---|---|
| *GDP growth* | | | |
| Observed | 0.25 | 0.59 | 1.2 |
| Counterfactual | 1.42 | 1.29 | 1.28 |
| *Inflation* | | | |
| Observed | 0.8 | 0.17 | 0.13 |
| Counterfactual | 0.91 | 0.86 | 0.81 |
| *Interest rate* | | | |
| Observed | 0.13 | 0.08 | 0.02 |
| Counterfactual | 0.22 | 0.24 | 0.26 |

Table 6 evidences that in the counterfactual the SW baseline generates a large response of GDP growth and inflation to a relatively small change in the short term interest rate. This effect is what

Del Negro et al. (2015) call the "Forward Guidance Puzzle". The scenario that the model captures so poorly the effects of forward guidance is a positive result in favor of the SW with expectations which perform well for forecasting key macroeconomic variables.

# 7 Concluding Remarks

This paper assesses the effectiveness of the forward guidance undertaken by European Central Bank using a standard medium-scale DSGE model à la Smets and Wouters (2007). Exploiting data on expectations from surveys, we show that incorporating expectations should be crucial in performance evaluation of models for the forward guidance. We conduct an exhaustive empirical exercise to compare the pseudo out-of-sample predictive performance of the estimated DSGE model with a Bayesian VAR and a DSGE-VAR models. DSGE model with expectations outperforms others for inflation; while for output and short term-interest rate the DSGE-VAR with expectations reports the best prediction. A counterfactual experiment suggests us that embodying expectations in a DSGE model framework is valid instrument to forecast the key macroeconomic variables.

# References


[1] Bäurle G., Kaufmann D., 2014. Exchange rate and price dynamics in a small open economy - the role of the zero lower bound and monetary policy regimes, Working Papers, 2014-10, Swiss National Bank.

[2] Banbura M, Giannone D, Reichlin L, 2010. Large Bayesian vector auto regressions, Journal of Applied Econometrics, John Wiley & Sons, Ltd., vol. 25(1), pages 71-92.

[3] Bekiros SD, Paccagnini A, 2014. Bayesian forecasting with small and medium scale factor-augmented vector autoregressive DSGE models, Computational Statistics & Data Analysis, Elsevier, vol. 71(C), pages 298-323.

[4] Bekiros SD, Paccagnini A, 2016. Policy-Oriented Macroeconomic Forecasting with Hybrid DGSE and Time-Varying Parameter VAR Models, Journal of Forecasting, Volume 35, Issue 7, November 2016, Pages 613–632.

[5] Brzoza-Brzezina, Michal and Kot, Adam, 2008. The Relativity Theory Revisited: Is Publishing Interest Rate Forecasts Really So Valuable?,MPRA Working Paper No. 10296, Munich Personal RePEc Archive.

[6] Coenen G, Straub R, Trabandt M, 2012. Fiscal Policy and the Great Recession in the Euro Area, American Economic Review, Papers & Proceedings, American Economic Association, vol. 102(3), pages 71-76, May.

[7] Cole, S. J., Milan, F., 2016. The Misspecification of Expectations in New Keynesian Models: A DSGE-VAR Approach, mimeo.

[8] D'Amico, S & T King, 2015. What Does Anticipated Monetary Policy Do?, Federal Reserve Bank of Chicago Working Paper N. 2015-10.

[9] Del Negro M, Schorfheide F, 2004. Priors from General equilibrium Models for VARs, International Economic Review, 45, 643-673.

[10] Del Negro, M., and S. Eusepi, 2011. Fitting observed inflation expectations, Journal of Economic Dynamics and Control, 35(12): 2105–2131.

[11] Del Negro, Marco, Marc Giannoni, & Christina Patterson, 2013. The Forward Guidance Puzzle, Federal Reserve Bank of New York, Staff Report No. 574.

[12] Eggertsson G. & M. Woodford, 2003. The Zero Interest-Rate Bound and Optimal Monetary Policy, Brookings Papers on Economic Activity, Spring.

[13] Eusepi, S. and B. Preston. 2011. Expectations, Learning, and Business Cycle Fluctuations, American Economic Review, 101(6): 2844-72.

[14] Filardo, A. & Hofmann, B., 2014. Forward guidance at the zero lower bound, BIS Quarterly Review, Bank for International Settlements, March.

[15] Ingram B, Whiteman C (1994) Supplanting the Minnesota Prior - Forecasting Macroeconomics Time Series using Real Business Cycle Model Priors, Journal of Monetary Economics, 34, 497-510

[16] Kolasa M, Rubaszek M, 2014. Forecasting with DSGE models with financial frictions, International Journal of Forecasting, forthcoming.

[17] Kolasa M, Rubaszek M, Skrzypczynski P, 2012. Putting the New Keynesian DSGE Model to the Real- Time Forecasting Test, Journal of Money, Credit and Banking, 44 (7), 1301–1324.

[18] Kool, C. J.M.; Middeldorp, M. & Rosenkranz, S., 2011. Central Bank Transparency and the Crowding Out of Private Information in Financial Markets, Journal of Money, Credit, and Banking, 43(4): 765-74.

[19] Krugman, P., 1998. It's Baaack: Japan's Slump and the Return of the Liquidity Trap. Brookings Papers on Economic Activity.

[20] McKay, A., E. Nakamura, & J. Steinsson, 2015. The Power of Forward Guidance Revisited, NBER Working Paper, No. 208

[21] Hirose, Y. & Inoue, A., 2015. The zero lower bound and parameter bias in an estimated DSGE model. Journal of Applied Econometrics. Forthcoming.

[22] Litterman RB (1981) A Bayesian Procedure for Forecasting with Vector Autoregressions, Working Paper, Federal Reserve Bank of Minneapolis.

[23] Litterman RB (1986) Forecasting with Bayesian Vector Autoregressions: Five Years of Experience, Journal of Business and Statistics 4(1), 25–38.

[24] Milani, F., 2007. Expectations, Learning, and Macroeconomic Persistance, Journal of Monetary Economics, 54 (7), 2065–2082.

[25] Milani, F., 2011. Expectation Shocks and Learning as Drivers of the Business Cycle, The Economic Journal, 121(552): 379–401.

[26] Ormeno A. & Molnar, K., 2015. Using Survey Data of Inflation Expectations in the Estimation of Learning and Rational Expectations Models, Journal of Money, Credit and Banking, 47(4): 673–699.

[27] Paccagnini, A. , 2010. DSGE Model Validation in a Bayesian Framework: an Assessment, Munich Repec, 24509.

[28] Paccagnini, A., 2011. DGSE Model Evaluation and Hybrid Models: A Comparison, EUI Max Weber Working Paper, 2011/11.

[29] Neri S., Notarpietro A. 2014. Inflation, debt and the zero lower bound, Questioni di Economia e Finanza (Occasional Papers), 242, Bank of Italy.

[30] Sims, C.A., 2002. Solving Linear Rational Expectations Models. Computational Economics, 20(1):1-20.

[31] Sims, C.A., 2007, Comment on On the Fit of New Keynesian Models, Journal of Business Economics and Statistics, 2007, Volume 25, Pages 152-154.

[32] Sims, C.A. & T. Zha, 1998. Bayesian Methods for Dynamic Multivariate Models, International Economic Review, 1998, 39(4):949-68.

[33] Slobodyan, S. & R. Wouters, 2012. Learning in an estimated medium-scale DSGE model. Journal of Economic Dynamics and Control, 36(1): 26–46.

[34] Smets, F. & Wouters, R., 2005. Comparing shocks and frictions in US and euro area business cycles: a Bayesian DSGE Approach. J. Appl. Econ., 20: 161–183.

[35] Smets, F. & Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97(3): 586-606.

[36] Theil H, Goldberg AS (1961) On Pure and Mixed Estimation in Economics, International Economic Review, 2, 65-78.

[37] Woodford, Michael. "Methods of Policy Accommodation at the Interest-Rate Lower Bound," in The Changing Policy Landscape. Proceedings of the symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, WY, August 30-September 1, 2012. Kansas City, MO: Federal Reserve Bank of Kansas City, 2012, pp. 185-288.

# Appendix

## A Sketch of the model

We consider a standard medium scale economy[11] *à la* Smets and Wouters (2007), which contains both nominal and real frictions affecting the choices of households and firms.

The aggregates resource constraints is given by

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{\imath}_t + u_y \hat{u}_t + \hat{\varepsilon}_t^g \tag{11}$$

where $c_y = c_*/y_*$, $i_y = i_*/y_*$ and $u_y = z_*^k k_*/y_*$ represent respectively the steady-state consumption, investment and rental rate of capital. The output produces is absorbed by consumption, $\hat{c}_t$, investment, $\hat{\imath}_t$, capital utilization rate $\hat{u}_t$, and an exogenous government spending shock $\hat{\varepsilon}_t^g$ that follows:

$$\hat{\varepsilon}_t^g = \rho_g \hat{\varepsilon}_{t-1}^g + \eta_t^g \sim N\left(0, \sigma_g^2\right)$$

A continuum of households of mass unity populate the economy with identical preferences that depends on hours worked and consumptions. Their behavior is captured by the following Euler equation:

$$\widehat{c}_t = c_1 \widehat{c}_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (h_t - E_t h_{t+1}) - c_3 \left(r_t - E_t \pi_{t+1} + \varepsilon_t^b\right), \tag{12}$$

where $c_1 = \frac{b\gamma}{1+b\gamma}$, $c_2 = \frac{(\sigma_c - 1)(w_* h_*/c_*)}{\sigma_c(1+b\gamma)}$ and $c_3 = \frac{1-b\gamma}{\sigma_c(1+b\gamma)}$. Consumptions $c_t$ is affected by the presence of external habits parameter $0 < b < 1$ and by the real interest rate. Parameter $\sigma_c$ refers to the degree of intertemporal elasticity of substitution while the parameter $\gamma$ captures the steady state growth rate. The term $\varepsilon_t^b$ is a preference shock affecting the subjective discount factor following an AR(1) process of the type:

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + \eta_t^b \quad \eta_t^b \sim N\left(0, \sigma_b^2\right)$$

Households can move resources between periods by purchasing one period bonds and renting capital to firms. Households make a capital accumulation decision and decide how many units of capital services to rent firms. The accumulation of capital, $\hat{k}_t$, is not only a function of the flow of investment, $\hat{\imath}_t$, but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance, $\widehat{\varepsilon}_t^i$:

$$\hat{k}_t = k_1 \hat{k}_{t-1} + (1 - k_1) \widehat{\imath}_t + k_2 \widehat{\varepsilon}_t^i, \tag{13}$$

[11] All variables are log-linearized around their steady state balanced growth path.

where $k_1 = \frac{(1-\delta)}{\gamma}$ and $k_2 = [1 - \frac{(1-\delta)}{\gamma}]\left(1 + \beta\gamma^{1-\sigma_c}\right)\gamma^2\varphi$.

Capital adjustment is costly and it is a function of the change in investment. The optimal investment choice is described by the investment Euler equation:

$$\hat{\imath}_t = i_1\hat{\imath}_{t-1} + (1 - i_1)\, E_t\hat{\imath}_{t+1} + i_2\widehat{q}_t + \widehat{\varepsilon}_t^i \tag{14}$$

where $i_1 = \frac{1}{(1+\beta\gamma^{1-\sigma_c})}$ and $i_2 = \frac{i_1}{\gamma^2\varphi}$. The parameter $\varphi$ is the elasticity of investment adjustment costs and $e_t^i$ is an investment-specific technology shock following an AR(1) process with $\rho_i$ the AR(1) coefficient

$$\widehat{\varepsilon}_t^i = \rho_i\widehat{\varepsilon}_{t-1}^i + \eta_t^i \quad \eta_t^i \sim N\left(0, \sigma_i^2\right)$$

The corresponding $\hat{q}_t$ equation measures the shadow price of a unit of investment good and takes the form of

$$\hat{q}_t = q_{1r}E_t\widehat{q}_{t+1} + (1 - q_{1r})\, E_t\widehat{z}_{t+1}^k - (\widehat{r}_t - \widehat{\pi}_{t+1}), \tag{15}$$

where $q_{1r} = \frac{(1-\delta)}{z_*^k+(1-\delta)}$ and $\delta$ is the depreciation rate of capital.

A labor union differentiates labor and sets wages in a monopolistically competitive market. Competitive labor packers buy labor services from the union, package and sell them to intermediate goods firms. Moreover, wages are staggered à la Calvo (1983). Union $j$ receives permission to optimally reset the nominal wage with probability $(1 - \xi_w)$. Therefore, salary is set according to:

$$\widehat{w}_t = w_1\widehat{w}_{t-1} + (1 - w_1)\left(E_t\widehat{w}_{t+1} + E_t\widehat{\pi}_{t+1}\right) - w_2\widehat{\pi}_t + w_3\widehat{\pi}_{t-1} - w_4\widehat{\mu}_t^w + \widehat{\varepsilon}_t^w \tag{16}$$

where $w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}$, $w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, $w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}$ and $w_4 = \frac{\left(1+\xi_w\beta\gamma^{1-\sigma_c}\right)(1-\xi_w)}{(1+\beta\gamma^{1-\sigma_c})\xi_w[(\phi_w-1)e^w+1]}$. The parameter $\beta$ represents the households discount factor, $\xi_w$ indicates the Calvo probability of not adjusting nominal wages, $\iota_w$ denotes the degree of wage indexation of non-adjusting unions, $(\phi_w - 1)$ is the steady state labor market markup, and $\widehat{\varepsilon}_t^w$ is the curvature of the Kimball aggregator in the labor market which takes the form of an AR(1) process:

$$\widehat{\varepsilon}_t^w = \rho_w\hat{\lambda}_{t-1}^w + \eta_t^w \quad \eta_t^w \sim N\left(0, \sigma_w^2\right)$$

The wage mark-up is the difference between the real wages and the marginal rate of substitution between consumption and labor:

$$\widehat{\mu}_w = \widehat{w}_t - \left[\sigma_l\widehat{h}_t + \frac{1}{1-b}\left(\widehat{c}_t - b\widehat{c}_{t-1}\right)\right], \tag{17}$$

where $\sigma_l$ is the elasticity of labor supply with respect to the real wage.

On the supply side, output is produced by a monopolistically a competitive sector with producers facing price rigidities. The aggregate production function takes the form of a standard Cobb Douglas.

$$\hat{y}_t = \phi_p \left[\alpha \left(\widehat{k}_{t-1} + \widehat{u}_t\right) + (1-\alpha)\,\widehat{h}_t\right] + \hat{\varepsilon}_t^a \tag{18}$$

i.e. output is produced using capital, labor and capital utilization, $\hat{u}_t$. $\hat{\varepsilon}_t^a$ is the transitory technology shock following an AR(1) process, $\rho_a$ is an autoregressive coefficient and $\mu_t^a \sim N(0, \sigma_a^2)$. The parameter $\phi_p$ represents one plus the share of fixed costs in production.[12], while $\alpha$ is the output elasticity to capital.

Cost minimization problem implies that:

$$\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t^k \tag{19}$$

where the degree of capital utilization is a positive function of the rental rate of capital:

$$\hat{r}_t^k = \frac{\sigma_u}{1-\sigma_u}\hat{u}_t \tag{20}$$

where $\sigma_u$ represents the positive function of elasticity of the capital utilization adjustment cost.

Due to price stickiness as in Calvo (1983) and partial indexation to lagged inflation of those prices that can not be re-optimised profits maximization by price-setting firms lead to the following New-Keynesian Phillips curve

$$\hat{\pi}_t = \pi_1 \hat{\pi}_{t-1} + \pi_2 \hat{\pi}_{t+1} - \pi_3 \widehat{\mu}_t^p + \widehat{e}_t^p, \tag{21}$$

where $\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_3 = \frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)\left(1-\xi_p\right)}{(1+\beta\gamma^{1-\sigma_c}\iota_p)\xi_p\left[\left(\phi_p-1\right)e^p+1\right]}$. $\iota_p$ represents the indexation parameter, $\xi_p$ the degree of price stickiness in goods market and $\widehat{e}_t^p$ is the curvature of Kimball aggregator in the goods market. The price markup disturbance follows an ARMA(1,1) process, $\widehat{e}_t^p = \rho_p \widehat{e}_{t-1}^p + \varepsilon_t^p - \mu_p \varepsilon_{t-1}^p$, $\rho_p$ is the AR(1) coefficient and $\varepsilon_t^p \sim N(0, \sigma_p^2)$. The term $\left(\phi_p - 1\right)$ is the steady-state markup in the goods market.

Marginal costs, $\widehat{mc}_t$, are affected by the factors costs and a productivity shocks

$$\widehat{mc}_t = -\hat{\varepsilon}_t^a + \alpha \hat{r}_t^k + (1-\alpha)\,\hat{w}_t \tag{22}$$

where the total factor productivity shocks follows an AR(1) process:

$$\hat{\varepsilon}_t^a = \rho_a \hat{\varepsilon}_{t-1}^a + \eta_t^a \sim N(0, \sigma_a^2)$$

[12] Note that from the zero profit condition in steady state, $\phi_p$ also represents the value of the gross steady state price markup.

The monetary authority sets the short-term interest rate according to a Taylor rule of the form:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r)\left[\phi_\pi \hat{\pi}_t + \phi_y \left(\hat{y}_t - \hat{y}_t^p\right)\right] + \phi_{\Delta y}\left[\left(\hat{y}_t - \hat{y}_t^p\right) - \left(\hat{y}_{t-1} - \hat{y}_{t-1}^p\right)\right] + \hat{\varepsilon}_t^r, \tag{23}$$

where $\hat{y}_t^p$ represents the level of output that would prevail under flexible prices and wages, $\phi_R$, $\phi_\pi$, $\phi_y$, $\phi_{\Delta y}$ are policy parameters referring to interest-rate smoothing, and the responsiveness of the nominal interest rate to inflation, to the output gap and to changes in the output gap, respectively.

$$\hat{\varepsilon}_t^r = \rho_r \hat{\varepsilon}_{t-1}^r + \eta_t^r \sim N(0, \sigma_r^2)$$

We model is solved assuming that agents have perfect knowledge about the model, its parameters and the true stochastic processes of the economy. Using the approach laid out in Sims (2002), we can write the model mapping the expectational errors into the set of structural shocks:

$$X_t = \Gamma X_{t-1} + \Omega \Sigma_t \tag{24}$$

where $X_t$ is a vector containing the endogenous variables of the model, $\Sigma_t$ is the vector of the exogenous shocks, and matrices $\Gamma$ and $\Omega$ contain the non-linear combinations of the model parameters. which yields the transition equation for our state space model. Note that 24 yields the transition equation for our state space model.

We estimate the preference shock, the technology shock, the wage and price markup shock, the monetary shock, government spending shock and the investment-specific technology shock. The innovations to these processes are structural shocks driving the model dynamics.

# B Data description

| Variable | Name | Source | Transformation |
|---|---|---|---|
| $y$ | GDP at market prices | ECB Real-time DB | Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator |
| $c_l$ | Final consumption expenditure | ECB Real-time DB | Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator |
| $i$ | Gross fixed capital formation | Eurostat | Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator |
| $\pi$ | HICP | Eurostat | - |
| $w$ | Compensation for employees | Eurostat | Log-difference; calendar and seasonally adjusted, converted in real term using GDP deflator |
| $r$ | Euribor | Eurostat | Divided by 4 |
| e | Employment | Eurostat | HP detrendend |
| $E_t y_{t+1}^{obs}$ | One-year ahed real GDP growth expectations | SPF, ECB | - |
| $E_t \pi_{t+1}^{obs}$ | One-year-ahed inflation expectations | SPF, ECB | - |

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