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On Nash Equilibria in Speculative Attack Models

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Abstract

Since a fixed exchange rate regime is a fixed price system, there is no theoretical reason to presume that the foreign exchange market clears, particularly during a speculative attack. This paper shows that equilibria where we allow for the possibility of such corner solutions are a superset of the previously examined "market-clearing" equilibria. The timing of the balance-of-payments crisis is no longer predictable in the same sense – multiple equilibria exist even in the very simplest speculative attack model.

JEL classification # F31, E58

I. Introduction

The vast literature that analyzes speculative attacks on fixed exchange rates has restricted its attention to a "market-clearing" concept of equilibrium. In this literature models are closed by assuming that supply is equal to demand in all markets, at all times. However, in a fixed exchange rate system this need not be the case. In a fixed exchange rate regime the price of foreign currency is not allowed to adjust in response to excess demand. Therefore, during speculative attacks we cannot simply presume that the foreign currency market will clear. This paper analyzes the implications of the possibility of excess demand for foreign currency during speculative attacks in a very simple, but representative, balance-of-payments crisis model. Rather than employing the market-clearing concept of equilibrium which has previously been applied, this paper focuses attention on Nash equilibria, requiring that all agents respond optimally at all times, but not requiring that supply is necessarily equal to demand.

One of the contributions of Obstfeld's seminal (1986) speculative attack paper was the development of rigorous proof that there can be only a single equilibrium in the standard speculative attack model.[1] In doing so he notes that the task "... is easily done once a precise definition of 'equilibrium' is adopted" pp. 75. Obstfeld (and the subsequent literature) chooses a market-clearing concept of equilibrium by imposing uncovered interest parity in all states. This approach is certainly valid, but it is not a logical necessity.

It is important to note that the market-clearing equilibria that have been previously analyzed are also Nash equilibria. However, these equilibria are constructed by assuming that markets clear, and then verifying that the results are Nash. As such, the literature has examined only a subset of the possible Nash equilibria. This paper examines the whole range of subgame-perfect Nash equilibria and argues that these equilibria are more interesting and realistic than the market clearing equilibria. In fact, market-clearing speculative attacks are likely to be zero probability events: Theoretically possible, but extremely unlikely.

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[1]Obstfeld (1986) Theorem 1.

In the Nash equilibria all markets clear in almost all times. Supply is equal to demand in all markets and at all times except possibly during a speculative attack. During an attack speculators may demand more foreign currency than the central bank is willing to supply. It is this possibility that has been unexamined in the previous literature. It is a possibility which is rich with interesting implications. It implies that the dichotomy maintained in the literature between predictable speculative attacks and self-fulfilling expectations of speculative attacks is spurious. Multiple equilibria exist even in the very simplest model. Hence, speculative attacks are fundamentally unpredictable. Even in the very simplest, fully deterministic model, the timing of a speculative attack cannot be pinned down to a particular date. Even though the date of the speculative attack cannot be known with certainty, much can still be said about its timing. There may exist multiple equilibria, but these are confined to a bounded set of states. Therefore movements in fundamentals can help to put bounds on the timing of the speculative attack even if they cannot identify the date with certainty.

These richer conclusions arise because excess demand in the foreign exchange market during a speculative attack changes the nature of the speculators' problem. When speculators are certain that their demand will be satisfied, they choose between foreign and domestic securities based solely on whichever offers the highest expected one-period return. However, when investors' choices today affect their opportunities tomorrow, the investment decision takes on an added dimension.

If a speculator expects that during a speculative attack his demand for foreign currency will not be satisfied, he will increase his current foreign currency purchases. By purchasing foreign currency early, before it becomes unavailable, the speculator can ensure that he will have it later. People buy foreign currency and invest in foreign bonds not only for their expected interest earnings and to exploit expected exchange rate movements, but also in order to hoard foreign currency in anticipation of its later scarcity. Therefore the possibility of excess demand for foreign currency during a speculative attack implies that in the time leading up to the attack domestic and foreign securities are very different instruments. This means that the speculators' decisions about where to invest cannot be solved by simply comparing the expected current returns on domestic and foreign securities. Buying foreign bonds has the further advantage of securing access to foreign currency during a future speculative attack. The speculators' problem involves comparing the expected excess return on foreign bonds over and above the return on domestic bonds in the period of speculative attack, with the opportunity cost of acquiring foreign currency one period earlier.

If speculators expect that a speculative attack tomorrow will cause shortages of foreign currency, they will realize that the future return on foreign bonds will exceed the return on domestic bonds. This will cause them to hoard foreign currency today. They will continue hoarding foreign currency as long as the opportunity cost of doing so is less than the present value of the expected excess returns on foreign bonds during the coming speculative attack. The central issue for determining the Nash equilibria is whether the central bank will be willing or able to accommodate speculators' desired level of foreign currency hoarding, or whether the hoarding motive will itself cause a breakdown in the fixed exchange rate regime.

This paper derives the Nash equilibria a simple perfect-foresight speculative attack model. A non-stochastic version of Flood and Garber's (1984) linear reduced-form model is used to demonstrate the mechanism at work and basic implications as clearly as possible. The insights gained from this simple structure can be easily applied to a wide variety of more complicated and realistic models. It is shown that once we relax the overly restrictive assumption that the foreign exchange market clears during a speculative attack, the nature of the attack is significantly altered. Many of the empirically observed characteristics of speculative attacks are explained much more convincingly with the Nash equilibria than with the market-clearing equilibria in these models.

Section 2 develops the model and derives the standard market-clearing equilibrium. This section also shows why market-clearing speculative attacks are likely to be zero probability events. Section 3 analyzes the full range of subgame-perfect Nash equilibria. Section 4 concludes with a discussion of how the basic insights of this paper can be extended to a wide variety of models.

II. The Model and Market-Clearing Equilibrium

To highlight the mechanism at work, a non-stochastic version of Flood and Garber's (1984) linear discrete time model is used. This perfect-foresight reduced-form framework provides one of the most transparent speculative attack models. It permits an examination of the dichotomy maintained in the literature between speculative attacks driven by movements in fundamentals, and multiple equilibria. Obstfeld (1986) provides an elegant proof that multiple equilibria are impossible in this model. However, here it will be shown that rational and self-fulfilling expectations of a crisis are in fact possible. Obviously these different conclusions do not stem from any technical problems in Obstfeld's work. Rather they arise due to the richer interaction that arises when we relax the overly restrictive assumption that markets always clear.

Before analyzing the Nash equilibria, it is useful to consider the problem as it is usually presented, with a market-clearing equilibrium concept. The structure and equilibrium analyzed here will follow the original model, but the exposition will be slightly different in order to lay the groundwork for the derivation of the Nash equilibria. This somewhat different presentation also allows an intuitive understanding of why there can be only a single market-clearing equilibrium in this framework, and why market-clearing equilibria are likely to be zero-probability events.

This framework posits a central bank with limited reserves which is pursuing a fixed exchange rate policy but also has other higher-priority policies which are fundamentally inconsistent with the exchange rate policy in the long run. It is a single good, small open economy model in which purchasing power parity implies that,

$$P_t = S_t P_t^* \qquad (1)$$

at all times t. S_t is the domestic currency price of foreign currency, P_t is the domestic price level, and P_t^* is the foreign currency price of output which is normalized to 1. There are four assets in this economy: Domestic money, domestic bonds, foreign currency and foreign bonds. Money demand is given by,

$$M_t^d / P_t = \alpha - \beta i_t \tag{2}$$

where i_t is the nominal interest rate on domestic securities. It is assumed that money demand is positive when the domestic interest rate is equal to the nominal interest rate on foreign securities i^*, so $(\alpha - \beta i^*) > 0$. i^* is assumed to be constant. The money supply is given by,

$$M_t^s = R_t + D_t \tag{3}$$

where R_t is the book value of central bank foreign currency reserves and D_t represents central bank holdings of domestic credit. The evolution of domestic credit follows the exogenous process,

$$D_t = D_{t-1} + \mu \tag{4}$$

where μ is a positive constant, usually interpreted as a need for the central bank to monetize a fiscal deficit.

As long as the central bank's foreign currency reserves are above zero it pursues a fixed exchange rate policy, buying and selling foreign currency on demand at the exchange rate $\overline{S}$. Once its reserves fall to zero, the central bank leaves the foreign exchange market forever and the exchange rate floats freely.[2] Taking the behavior of the central bank as exogenous is clearly problematic. A promising line of research on managed exchange rates aims at relaxing this assumption.[3] However, while the point of this paper is equally applicable to a framework with an optimizing central bank, it can be seen most clearly by focusing attention on the decisions of speculators. Therefore the traditional assumption of exogenous central bank behavior is maintained.

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[2] Assuming that zero is the critical level of reserves where the central bank abandons the fixed exchange rate regime is arbitrary. This assumption is made solely to facilitate direct comparison between the Nash equilibria derived in this paper and Flood and Garber's original work. The results are unaltered by a more general specification.

[3] Models emphasizing strategic interaction between speculators and the central bank in response to fundamentals include Andersen (1994), Chen (1995), Ozkan and Sutherland (1994 and 1995), and Pastine (2000 and 2002a). Models emphasizing the possibility that strategic interaction can lead to multiple equilibria include Davies and Vines (1995), Obstfeld (1994 and 1996), Ozkan and Sutherland (1998), and Velasco (1997).

To analyze the incentives of speculators, define the shadow floating exchange rate, $\tilde{S}_t$, as the exchange rate that would prevail if the exchange rate was floating at t, and reserves were therefore zero. It is assumed that once the fixed exchange rate system breaks down the exchange rate will float freely thereafter without restrictions on capital movements. Therefore uncovered interest parity implies,

$$i_t = i^* + E_t [(S_{t+1}/S_t) - 1] \qquad (5)$$

Combining this with (1)-(4) yields,

$$[(\alpha-\beta i^*)+\beta]\, \tilde{S}_t = D_t + \beta\, E_t[\, \tilde{S}_{t+1}] \qquad (6)$$

Assuming no speculative bubbles in the floating rate period this difference equation implies,

$$\tilde{S}_t = \frac{D_t}{(\alpha-\beta i^*)} + \frac{\beta\mu}{(\alpha-\beta i^*)^2} \qquad (7)$$

At this point in the analysis, the usual argument is that competition for speculative profits will cause the breakdown in the fixed exchange rate regime as soon as profit opportunities arise. That is, there must be a speculative attack as soon as the shadow floating exchange rate rises to the fixed rate, $\tilde{S}_t=\overline{S}$. Here it will prove more enlightening to present the equilibrium in an equivalent, but slightly different manner. In this model there are two important financial markets: The market for domestic bonds, and the market for foreign currency. The approach will be to examine the interest rate required to clear each of these markets *during a speculative attack*. It will then be shown that there can only be one time when both markets can be in equilibrium during an attack, and that point is where $\tilde{S}_t=\overline{S}$.

If there is a balance-of-payments crisis in period t then R_t=0 and equilibrium in the domestic money market and goods market requires that (1)-(3) hold,

$$i_t = (\alpha/\beta) - D_t / (\overline{S}\beta) \qquad (8)$$

The domestic interest rate that would clear domestic markets *in the event of an attack* is decreasing in t, since D_t is increasing in t. In the absence of a speculative attack, expanding domestic credit puts downward pressure on the domestic interest rate, so investors move assets overseas until the foreign and domestic interest rates are equalized. Therefore, expanding domestic credit implies that prior to the speculative attack, central bank reserves are declining. With lower central bank reserves, smaller speculative attacks will cause a breakdown in the fixed exchange rate regime. Smaller speculative attacks will result in smaller contractions in the money supply. Hence, expanding domestic credit implies that in later dates lower interest rates are required to clear the money market during a speculative attack. This is what is shown in equation (8). It is important to realize that this is not the time path of the interest rate. It says only that if there is a speculative attack at time t, the domestic interest rate will be given by (8).

A similar exercise can be done for the foreign exchange market. If there is a speculative attack in period t, (5) will hold and agents will expect that in the future the exchange rate will be floating, so (7) will hold. This implies,

$$i_t = i^* - 1 + \frac{\left[(\alpha - \beta i^*) + \beta\right]\mu}{\bar{S}\,(\alpha - \beta i^*)^2} + \frac{D_t}{\bar{S}\,(\alpha - \beta i^*)} \tag{9}$$

Since domestic credit is increasing in t, the shadow floating exchange rate is also increasing in t. Therefore the return on foreign assets is higher during later abandonments of the fixed exchange rate regime, due to the higher post-abandonment floating exchange rate. Hence the domestic interest rate required to compensate investors for this forgone profit is increasing in t. Once again, this is not the time path of the interest rate. It is simply the interest rate that will clear the foreign exchange market *in the event of a speculative attack* in time t.

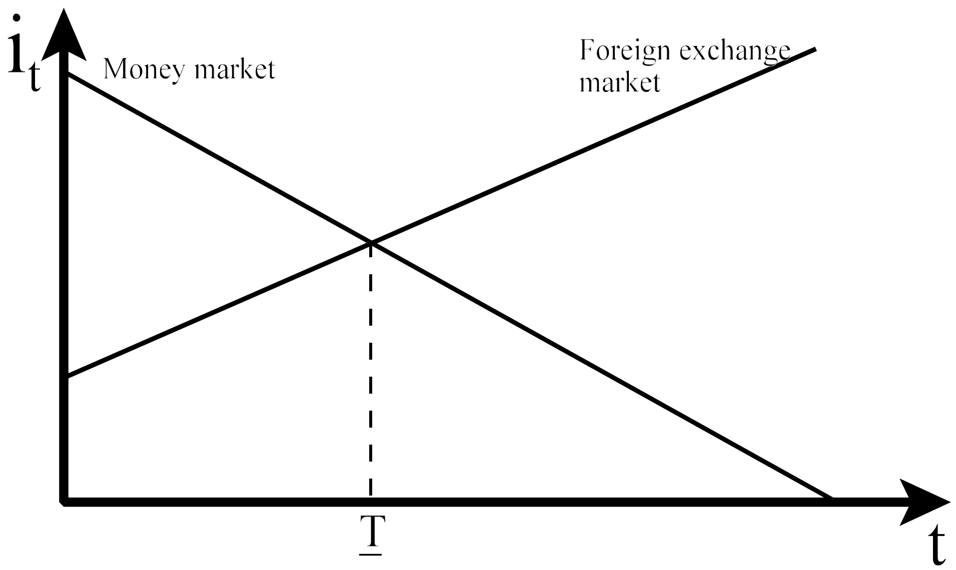

Figure 1: Domestic interest rate required to equate supply and demand during a speculative attack. (not time paths)

Figure 1 shows (8) and (9), the domestic interest rate required to equate supply and demand in the domestic money market and foreign exchange market respectively during a speculative attack. It is also worth noting that these also represent the expected return on domestic securities and foreign securities during a speculative attack. Define $\underline{T}$ as the time where (8) and (9) are equal, the time where supply and demand will be equal in all markets during a speculative attack. It is straightforward to verify that $\underline{T}$ is also the time where $\tilde{S}_t=\bar{S}$, so $\underline{T}$ is the date of the speculative attack in the traditional analysis.

Before $\underline{T}$, the post-attack floating exchange rate would be less than the fixed exchange rate ($\tilde{S}_t<\bar{S}$) so no individual would find it profitable to join in an attack on central bank reserves and therefore an attack cannot occur. $\underline{T}$ is the first time where a balance-of-payments crisis is possible. In the event of a balance-of-payments crisis at $\underline{T}$ all markets would clear, so a crisis at the first time $t\geq\underline{T}$ is a potential equilibrium outcome of this game, as has been identified in the previous literature.

Several interesting conclusions emerge from Figure 1. The most obvious is that there can be only one time ($\underline{T}$) where all markets clear during a speculative attack. This is in accordance with the traditional results of these models.[4] But this observation immediately casts doubt whether it is reasonable to assume that all markets clear during a speculative attack. It is only possible for all markets to clear during an attack if $\underline{T}$ is hit *exactly*. In general, this will not happen. In non-deterministic versions of the model, domestic credit evolves stochastically. This is equivalent to "time" evolving stochastically in this framework. This implies that hitting $\underline{T}$ exactly is a zero-probability event. Hence, it is not probable that all markets clear during speculative attacks.

The literature has typically sidestepped this problem of hitting $\underline{T}$ precisely by simply assuming that the speculative attack occurs as soon as $t \geq \underline{T}$. However, this implies that during the speculative attack the domestic money market and the foreign exchange market cannot both be in equilibrium. This leads to a strange state of affairs where the equilibrium is derived by assuming that all markets clear, but where the ensuing equilibrium results in some markets not clearing.

III. Nash Equilibrium

Since market clearing is a zero-probability event, and since in a fixed-price system there is no theoretical reason to presume that markets clear, it seems reasonable to treat the possibility of shortages of foreign currency during the speculative attack explicitly.

[4]See in particular Obstfeld (1986) for an elegant proof that there can be only one equilibrium in a generalized version of this model. As was done here, Obstfeld assumes that all markets clear by imposing uncovered interest parity in all states.

A. Investment Returns

The domestic interest rate is always set in the domestic money and bond markets. These markets will always clear since prices in these markets are free to adjust in response to excess demand. However, if a crisis occurs after $\underline{T}$, then the interest rate set in these markets will be too low to clear the foreign exchange market. The relatively high expected return on foreign securities would induce speculators to demand more foreign currency than the central bank would be willing to supply. However, since the price of foreign currency is fixed, this does not imply that speculative attacks during these later times are not Nash equilibrium outcomes. All that is required for Nash equilibrium is that each speculator behaves optimally given the behavior of the central bank and the behavior of other speculators.

The optimal strategy for a speculator depends crucially on the speculator's expectations of the future. For $t>\underline{T}$, if a speculator expects an attack in the next period, he will realize that foreign currency will be rationed. This will give him an incentive to purchase foreign currency before it becomes unavailable. In the period before the expected speculative attack, each speculator will compare the expected benefit of acquiring foreign currency early with the opportunity cost of doing so. Hence, the analysis of this problem requires a specification of speculators' opportunity cost of hoarding foreign currency. Here, for clarity, I will take the very simplest specification of speculator opportunity costs: The forgone pecuniary returns from other investment opportunities.[5]

If an individual holds foreign bond during a speculative attack he earns an expected return of $\{i_t^* + E_t[(S_{t+1}/S_t) - 1]\}$ where $E_t[S_{t+1}]$ is given by (7). So this expected return is equivalent to the right-hand side of (9). Therefore, the foreign exchange market equilibrium line in Figure 1 can be reinterpreted as the expected return on holding foreign securities during a speculative attack.

[5]The reduced form nature of the model makes it difficult to say precisely what specification of speculator opportunity cost is consistent with the model. In any case, this very simple form for the opportunity cost illustrates the mechanism at work in the most transparent manner. Section 4 discusses how the opportunity cost of holding foreign currency can be derived more generally.

Since the money market and the domestic bond market are always in equilibrium, (1)-(3) must hold so,

$$i_t = (\alpha/\beta) - [\, R_t + D_t \,] / (\, \overline{S}\beta) \qquad (10)$$

Therefore (8) can still be interpreted as the return on domestic securities during a speculative attack. However, it also takes on an additional interpretation. Equation (10) always holds in the fixed exchange rate regime, and the fixed exchange rate will not breakdown as long as foreign currency reserves are above zero. Together these imply that the fixed exchange rate regime will not breakdown as long as,

$$i_t < (\alpha/\beta) - D_t \,/\, (\, \overline{S}\beta) \qquad (11)$$

So the graph of (8) can be interpreted as the highest return on domestic securities consistent with continued maintenance of the fixed exchange rate. These interpretations are presented in Figure 2. $\underline{T}$ is still at the intersection where the return on domestic and foreign securities are equalized during a speculative attack.

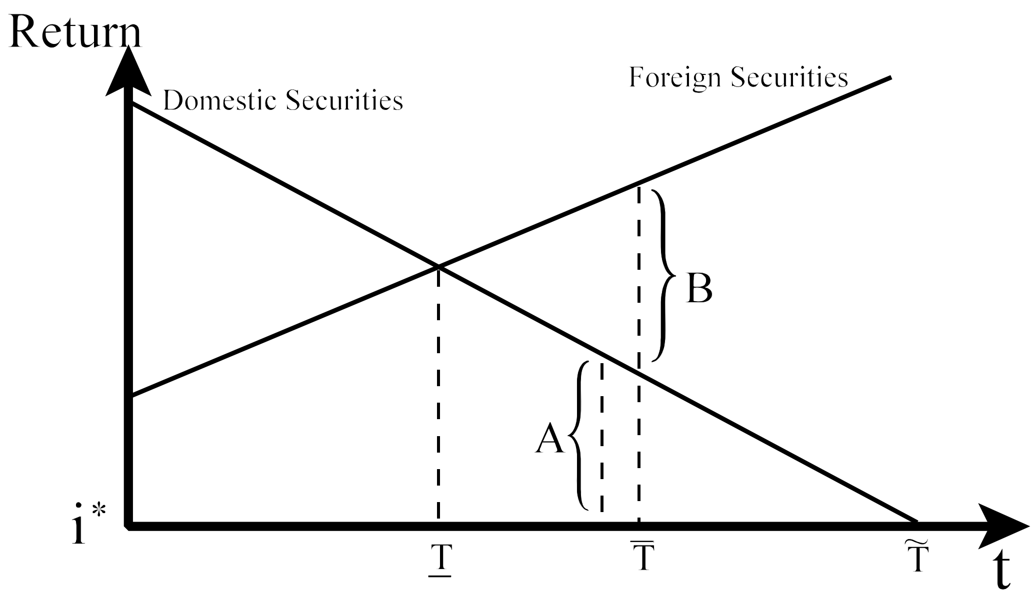

Figure 2: Reinterpreting Figure 1 – Return on investments during a speculative attack. $\overline{T}$ defined by A=δB.

It will be shown that hoarding before an expected attack in any time before a critical time $\overline{T}$ will not cause an earlier breakdown in the fixed exchange rate regime. Expectations of an attack anywhere in the range $[\underline{T}, \overline{T})$ are rational. Between $\underline{T}$ and $\overline{T}$-1 it is rational for a speculator to attack the central bank's foreign currency reserves if he expects that others will as well, but it is also rational not to attack if he believes that others will not attack. Between these two dates speculators can form rational and self-fulfilling expectations of a crisis.

The time of the critical period $\overline{T}$ will be examined, and the subgame-perfect Nash equilibria will be constructed, by backward induction. To do so it is first necessary to establish an end to the game, a time $\tilde{T}$ where the fixed exchange rate will be abandoned with certainty, if it has not already been abandoned. Momentarily, suppose that speculators mistakenly expected that the central bank would never abandon the fixed exchange rate regime, $E_t[S_{t+1}]=\overline{S}$ for all t. In that case there would not be a speculative attack so (5) would hold, implying that $i_t=i^*$ for all t. So, from (1)-(3),

$$R_t + D_t = (\alpha - \beta i^*)\,\overline{S} \qquad (12)$$

Since D_t is increasing, R_t must be decreasing and the fixed exchange rate will be abandoned at the date $\tilde{T}$ defined by,

$$D_{\tilde{T}} = (\alpha - \beta i^*)\,\overline{S} \qquad (13)$$

The intuition for this is straightforward. When speculators do not expect a change in the fixed exchange rate regime, they will adjust their portfolios until the return on foreign and domestic securities are equalized. Expanding domestic credit puts downward pressure on the domestic interest rate resulting in increasing holdings of foreign securities, and hence decreasing central bank reserves. This implies that eventually reserves will hit their lower bound, resulting in a breakdown of the fixed exchange rate regime. Hence, at this point a domestic interest rate equal to the foreign interest rate will clear the domestic money market during a breakdown in the fixed exchange rate regime. Since equation (8) represents the domestic interest rate required to clear the domestic money market when reserves are zero, time $\tilde{T}$ is where (8) equals the interest rate on foreign securities.

From this, the subgame-perfect Nash equilibria can be constructed by backward induction. At time $\tilde{T}$ the fixed exchange rate must be abandoned with certainty, if it has not already been abandoned. Given this information, we can compute speculators' optimal strategies in $\tilde{T}$-1, and from these we can compute their optimal strategies in $\tilde{T}$-2, and so on.

B. Hoarding

Speculators know that at $\tilde{T}$ the fixed exchange rate will be abandoned with certainty if it has not already been abandoned, and they know that there will be high returns in the foreign securities market. However, they also know that foreign currency will be unavailable in $\tilde{T}$, since at $\tilde{T}$ reserves are zero even before speculators attempt to exploit the devaluation. So in period $\tilde{T}$-1 they will consider purchasing foreign currency and holding it one period. If they do this in aggregate, however, they will raise the period $\tilde{T}$-1 interest rate which will make domestic securities relatively more attractive in $\tilde{T}$-1. They will continue to purchase foreign securities until the discounted gain from holding foreign securities at $\tilde{T}$ is equal to the opportunity cost of holding them in period $\tilde{T}$-1. The opportunity cost of holding foreign securities is just the forgone interest from holding domestic securities. If this domestic interest rate in $\tilde{T}$-1 is low enough that (11) holds then hoarding does not itself lead to a breakdown of the fixed exchange rate, implying that the expectations of no breakdown until $\tilde{T}$ were rational. On the other hand, if this domestic interest rate (the opportunity cost) is so high that it violates (11), then the hoarding itself will lead to a breakdown in period $\tilde{T}$-1.

$\overline{T}$ is the last date where the expectation of no earlier breakdown is rational. At any time where a speculative attack is expected, the incentive to hold foreign securities is the difference between the return on foreign securities and domestic securities (the distance labeled B in Figure 2). In the period before, the maximum return on domestic securities that can be sustained without a breakdown of the fixed exchange rate is given by the domestic securities line (A in Figure 2).

Defining $\delta \in (0,1)$ as the one period discount rate, and assuming that speculators are risk neutral,[6] $\overline{T}$ is the time where $A=\delta B$. At $t \geq \overline{T}$ the discounted gain from hoarding (δB) is so large that the interest rate in the previous period must rise above the maximum sustainable without a breakdown of the fixed exchange rate (A). Therefore it is not rational to believe that an attack in the period before will not occur. Before $\overline{T}$, $A<\delta B$ so the discounted gain from hoarding is not so great that if forces a speculative attack in the period before. So $\overline{T}$ is the first time where expectations of an attack will cause an even earlier attack.

Now consider the period $\overline{T}$-ε. All speculators will realize that the fixed exchange rate must be abandoned at $\overline{T}$-ε and that foreign currency supplies will not meet their demand at that date. So they purchase foreign securities in $\overline{T}$-ε-1, holding them in anticipation of the speculative attack in $\overline{T}$-ε. While this hoarding of foreign securities depletes the central bank's reserves in period $\overline{T}$-ε-1, they are still above zero and the exchange rate is not abandoned. Expectations of no speculative attack until $\overline{T}$-ε are rational. In fact, by the same argument, expectations of an attack anywhere in the range $[\underline{T}, \overline{T})$ are rational. Anywhere in that range it is worthwhile to join in an expected attack ($t \geq \underline{T}$) and hoarding in the period before will not precipitate an even earlier breakdown ($t<\overline{T}$).

C. The Timing of the Speculative Attack

Since the speculative attack must occur in the first period greater than or equal to $\overline{T}$-1, if it has not already occurred, the relationship between $\overline{T}$ and the other critical times $\underline{T}$ and $\tilde{T}$ is

[6] The risk neutrality assumption implies that the form of the rationing rule is immaterial. Analogous conditions can be derived for risk averse speculators under various rationing schemes.

of interest. To examine the timing of $\overline{T}$, the rather tedious algebra can be simplified considerably by the normalization $\tau \equiv t - \underline{T}$. From this define,

$$\begin{aligned} \underline{\tau} &\equiv \underline{T} - \underline{T} = 0 \\ \overline{\tau} &\equiv \overline{T} - \underline{T} \\ \tilde{\tau} &\equiv \tilde{T} - \underline{T} \end{aligned} \tag{14}$$

The crisis can only occur at $\underline{\tau}$ or later. And it must occur before $\overline{\tau}$. Thus, if $\overline{\tau}>1$ there will be periods when speculative attacks are rational, but where it is also rational not to attack. If $\overline{\tau}>1$ then there will exist a range of times where speculators can form rational and self-fulfilling expectations of a crisis. Note that (7) and $\tilde{S}_{\underline{T}}=\overline{S}$ give,

$$D_{\underline{T}} = (\alpha - \beta i^{*})\, \overline{S} - \beta\mu / (\alpha - \beta i^{*}) \tag{15}$$

so with (13) and (4) this implies that $\tilde{T}-\underline{T}= \beta/(\alpha-\beta i^{*})$ or $\tilde{\tau}=\beta/(\alpha-\beta i^{*})$.

Defining $A(\tau)$ as the return on domestic securities during a speculative attack at time τ, from (8) and (4),

$$A(\tau) = \frac{\alpha}{\beta} - \frac{D_{\underline{T}}}{\overline{S}\beta} - \frac{\mu\tau}{\overline{S}\beta} \tag{16}$$

and using (15),

$$A(\tau) = i^{*} + \frac{\mu}{\overline{S}\,(\alpha-\beta i^{*})} - \frac{\mu\tau}{\overline{S}\beta} \tag{17}$$

Define $B(\tau)$ as the excess return on foreign securities during a speculative attack at time τ, which is equal to (9) minus (8),

$$B(\tau) = i^{*} + \frac{\left[(\alpha-\beta i^{*}) + \beta\right]\mu}{(\alpha-\beta i^{*})^{2}\,\overline{S}} - \frac{(\alpha+\beta)}{\beta} + \frac{\left[(\alpha-\beta i^{*}) + \beta\right]D_{\underline{T}}}{(\alpha-\beta i^{*})\,\beta\overline{S}} + \frac{\left[(\alpha-\beta i^{*}) + \beta\right]\mu\tau}{(\alpha-\beta i^{*})\,\beta\overline{S}} \tag{18}$$

and using (15) this reduces to,

$$B(\tau) = \frac{\left[(\alpha-\beta i^{*}) + \beta\right]\mu\tau}{(\alpha-\beta i^{*})\,\beta\overline{S}} \tag{19}$$

Since $\bar{\tau}$ is defined by A(τ-1)=δB(τ),

$$\bar{\tau} = \frac{(\alpha-\beta i^*)\,\beta\bar{S}\,i^*}{\{(\alpha-\beta i^*) + \delta[(\alpha-\beta i^*)+\beta]\}\,\mu} + \frac{[(\alpha-\beta i^*)+\beta]}{(\alpha-\beta i^*) + \delta[(\alpha-\beta i^*)+\beta]} > 0 \qquad (20)$$

Since, if it has not already occurred, a speculative attack will occur with certainty in the first period greater than or equal to $\bar{\tau}$-1, the relationship between this date and the other critical dates is of interest.

$$\bar{\tau}-1 = \frac{(\alpha-\beta i^*)\,\beta\bar{S}\,i^*}{\{(\alpha-\beta i^*) + \delta[(\alpha-\beta i^*)+\beta]\}\,\mu} + \frac{\beta - \delta[(\alpha-\beta i^*)+\beta]}{(\alpha-\beta i^*) + \delta[(\alpha-\beta i^*)+\beta]} \qquad (21)$$

If $\bar{\tau}$-1>0 then there exists a set of times during which rational and self-fulfilling expectations can lead to a speculative attack. So equation (21) implies that, depending on parameter values, there may be a non-empty set of times during which rational and self-fulfilling expectations can lead to a speculative attack.

This is generally true, but can be seen more clearly if we make the natural assumption that the subjective rate of time preference is equal to the world interest rate, which yields $\delta = \frac{1}{(1+i^*)}$. In this case (21) implies that there are multiple equilibria if,

$$(1-\delta)\,\beta\bar{S} + \frac{(1-\delta)\,\beta\mu}{(\alpha-\beta i^*)} > \delta\mu \qquad (22)$$

For this, $(1-\delta)\beta\bar{S} \geq \delta\mu$ is a sufficient condition. So if the rate of expansion of domestic credit, μ, is sufficiently small, then their will be a range of times where speculators can form rational and self-fulfilling expectations of a crisis. The fact that domestic credit is expanding ensures that a crisis must come. But unless domestic credit is expanding very quickly, movements in fundamentals cannot pin it down to a precise time. Movements in fundamentals can, however, determine a bounded range of times during which the crisis must occur.

As in the market-clearing equilibrium, there can be no speculative attack before $\underline{T}$. The difference between the Nash equilibria and the market-clearing equilibrium arises if $\bar{T}$-1>$\underline{T}$, which will occur if the rate of expansion of domestic credit is not too great. Between these two times, a speculative attack at any date can be supported as a Nash equilibrium based on different sets of speculator expectations of the behavior of other speculators. In this range it is rational for speculators not to attack the central bank's foreign currency reserves if they believe that other speculators will not attack either. However, it is also rational to attack if speculators expect that others will as well. If it has not occurred before that time, a speculative attack will occur in the first period greater than or equal to the max[$\underline{T}$, $\overline{T}$-1].

Unless the speculative attack occurs precisely at $\underline{T}$, speculators will wish to purchase more foreign currency at the fixed exchange rate than the central bank is willing to supply. This excess demand for foreign currency is a result of the low fixed price of foreign currency offered by the central bank. Speculators can, of course, also purchase foreign currency in private markets. These markets would be charging the shadow floating exchange rate, and hence there would be no excess demand at the market price. During the speculative attack there will be two foreign exchange markets operating, each with a different price. It is in the controlled price market run by the central bank that there is excess demand for foreign currency.

If the attack occurs after $\underline{T}$, speculators will realize that there will be excess demand for foreign currency at the fixed exchange rate during the attack period. They also realize that there will be a devaluation and that this will result in excess returns on foreign securities during the attack. Therefore they will purchase foreign currency denominated bonds in the period before the attack, hoarding foreign currency in anticipation of its later scarcity. However, as they buy foreign exchange in aggregate there is a decline in the domestic supply of loanable funds. In the period before the attack, the increased holdings of foreign currency denominated bonds decreases the money supply. Hence the domestic interest rate must rise in order to induce speculators to hold domestic securities as well. This leads to increased domestic interest rates in the period before the attack, which deters speculators from holding too much foreign currency. So, in the period before the attack, the central bank is still able to maintain the fixed exchange rate.

It is worth noting that, in contrast to the market-clearing equilibrium, in the Nash equilibria agents are making expected profits at the central bank's expense. However, these profits are finite and cannot be increased by any alternative investment strategies. While speculators may make a profit on foreign securities during the speculative attack, in order to acquire more foreign currency they must incur a corresponding loss in the period before. In the period before the attack the domestic interest rate rises dramatically, so investors are indifferent between holding domestic bonds, with high interest rates, and holding foreign bonds which allow them to exploit the devaluation one period later.

Although in the period before the attack each individual speculator is indifferent between foreign and domestic bonds, the aggregate portfolio is uniquely determined in any given equilibrium. Any aggregate deviation from this portfolio will alter the domestic interest rate and individual speculators would no longer be indifferent. For example, suppose that aggregate holdings of foreign bonds were above their equilibrium level in the period before the attack. This aggregate movement away from domestic bonds would drive up the domestic interest rate, and individual speculators would find the high returns on domestic assets more than enough to compensate them for the return on foreign bonds during the future devaluation. Hence individuals would move their assets back to domestic securities.

IV. Conclusion

This paper analyzes Nash equilibria in speculative attack models. While the analysis is in terms of speculative attacks on fixed exchange rate regimes, the point is equally applicable to speculative attacks in other markets.[7] Previous work on speculative attacks has focused attention on a market-clearing concept of equilibrium, which by assumption does not permit excess demand in any state, even during a speculative attack. However, since a fixed exchange rate regime is a system of fixed prices, prices do not adjust in response to excess demand. Therefore there is no theoretical reason to presume that the foreign exchange market clears during a

[7]For example, speculative attacks on natural resource price fixing schemes, such as the gold standard analyzed in the seminal paper by Salant and Henderson (1978).

speculative attack. Moreover, the states of nature that permit all markets to clear during a speculative attack are likely to be zero probability events. In the Nash equilibria, however, agents respond optimally to their environment and to the behavior of others, but all markets are not required to clear in all states. This permits the possibility of shortages of foreign currency during speculative attacks.

While analysis is done in a very simple benchmark speculative attack model, the insights are easily extendable to a wide variety of frameworks. The essential insight is that during speculative attacks speculators may demand more foreign currency than the central bank is willing to supply. The possibility of excess demand can be incorporated into optimizing models by using the appropriate Kuhn-Tucker condition on speculators' foreign currency purchases, rather than the more restrictive first-order condition.[8] Subgame perfection is ensured by requiring rational forward-looking hoarding behavior in response to expected excess demand. Thus, in the period before an expected attack speculators must compare the opportunity cost of hoarding foreign currency with the expected future gains from having it. Typically, the opportunity cost and excess return calculus will be in terms of expected marginal utility, rather than the algebraically simpler pecuniary returns analyzed here, but conceptually the problem is identical.[9]

It should be noted that additional subgame-perfect Nash equilibria of the kind analyzed here will not exist in continuous-time models. In continuous time there is no opportunity cost of holding foreign currency. In these models, if speculators expect an attack they can acquire foreign currency an instant before the attack. Since the foreign currency would only need to be held for an infinitely short time, there is no finite domestic interest rate that could deter investors from purchasing an infinite amount of foreign currency in anticipation of even a small devaluation. Hence in continuous time rational hoarding will always cause an even earlier breakdown in the fixed exchange rate regime. Whether or not this is realistic depends on the

[8]The first-order condition has traditionally been imposed in these models, ensuring an interior solution, and hence market clearing.

[9]See Pastine (2002b) for an application to a model with explicitly optimizing speculators. This paper shows that incorporating the possibility of excess demand for foreign currency during speculative attacks can result in increased costs of consumption smoothing. This leads to previously unanalyzed real effects of speculative attacks.

institutional features of actual markets. In a world where investors are typically able to lock in high interest rates for 24 hours, there are substantial opportunity costs to forgoing high interest rate domestic investments. This paper shows that these opportunity costs can lead to equilibrium excess demand for foreign currency during speculative attacks, resulting in a much wider range of equilibria.

The existence of excess demand is notoriously difficult to determine empirically. Particularly when excess demand would only be present briefly, during a speculative attack, it seems unlikely that conclusive evidence one way or the other is likely to be forthcoming. However, suggestive evidence is available. It is very common for speculative attacks to be followed by a brief period where the central bank suspends foreign currency trading prior to the devaluation or abandonment of the fixed exchange rate regime. One reasonable interpretation of this phenomenon is that demand for foreign currency exceeds the amount the central bank is willing to supply, and that the central bank suspends foreign currency trading in order to suppress that excess demand. At an informal level, the air of panic that frequently accompanies speculative attacks is also suggestive of fear that demand might not be satisfied.

This paper shows that incorporating this possibility can help to improve our understanding of speculative attacks. Even in the very simplest, fully deterministic balance-of-payments crisis model, the crisis need not be confined to a particular date. Fundamentals do, however, allow for the determination of a bounded range of possible crisis dates. This range begins at the previously identified market-clearing equilibrium, and continues until excess demand during a speculative attack would be so great that rational hoarding in the period before would itself cause a breakdown in the exchange rate regime.

Closing the models by assuming Nash behavior, rather than by imposing market clearing, yields results which are closer to real-world observations of speculative attacks. In the market-clearing equilibrium the fixed exchange rate mechanism breaks down as soon as the shadow floating exchange rate is equal to the fixed rate. This means that any jump in the nominal exchange rate must be proportional to the size of the last shock. In fact, the jump in the nominal exchange rate must be less than or equal to the jump that would occur if the same shock hit in a flexible exchange rate regime. In the Nash equilibria, however, there is a possibility for sudden, large depreciations of the exchange rate. The fixed exchange rate system may persist for some time after the shadow floating rate rises above the fixed rate. This provides a possibility of substantial exchange rate misalignment, and therefore substantial exchange rate adjustment.

Closely related to this is the possibility for a sustained peso problem. The market-clearing equilibria can sustain a forward exchange rate premium in stochastic versions of these models. However, the premium can only last for a length of time proportional to the size of the shocks. This is because the premium stems from the probability of the next shock driving fundamentals over a critical threshold, inducing a crisis. In the Nash equilibria there is also the possibility of a period of times where rational and self-fulfilling expectations can induce a crisis. The forward rate could exceed the spot rate during this entire period, even in this non-stochastic version of the model. Therefore the length of the peso problem need not be proportional to the size of the shocks.

This representative agent framework implicitly assumes efficient rationing of foreign currency during the speculative attack. With other rationing mechanisms different speculators would get different amounts of foreign currency in the attack period, and thereafter they would no longer be identical. Other rationing schemes could also be analyzed. With proportional (random) rationing, for example, some consumers would not get to purchase any foreign currency during the attack. The consumption of these consumers would drop after the attack as their return on investment during the attack would be less than their a priori expected return. Other consumers, however, would be lucky enough to have all their demand for foreign currency satisfied. The consumption of these consumers would increase after the speculative attack, since their return on investment during the attack would have been higher than their a priori expected return. Analysis along these lines may provide a theoretical foundation for the common observation that some groups lose during speculative attacks, but that others gain. It may also provide interesting insights into the real effects of speculative attacks.

These issues introduce the possibility that speculative attacks may have serious distributional consequences. Future research could examine the nature of these distributional issues in order to understand the political economy of fixed exchange rates. With the Nash equilibria there are significant profits to be made if agents can get access to foreign currency at the fixed exchange rate during the speculative attack. If a speculator is able to purchase foreign currency during the attack period he will make substantial profits without incurring the opportunity cost of hoarding. This introduces considerable scope for rent-seeking competition for privileged access to foreign exchange. It may be enlightening to investigate the relationship between these issues and the institutional features of central bank trading policies.

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