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Population Health**

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Methods for Studying Dominance and Inequality in Population Health¹

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Abstract: This paper reviews methods for studying dominance and inequality in health economics. It concentrates on “pure inequality” as opposed to inequality which is related to income or some other measure of household resources. The paper reviews methods for cases when health can be measured cardinally and ordinally. There is also a brief review of statistical inference in this area.

Keywords: Dominance, Inequality.

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Methods for Studying Dominance and Inequality in Population Health

(1st Draft)

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1. Introduction

This chapter covers a number of measurement issues which arise in Health Economics. The first of these arise when economists wish to make comparisons between populations on the basis of some measure of health, h , where h_i refers to the value of the health measure for individual i . Such comparisons may be between different populations at the same point in time, or between the same population at different points in time, or indeed a combination of the two. In some cases it may be desirable to compare some measure of central tendency, such as the mean or median, \bar{h} . In some cases however, we may also be concerned about the distribution of the health measure. This may arise for example because the underlying individual utility function is increasing and concave in the health measure, h_i (presuming for the sake of exposition that a higher value of the health measure increases utility) or it may arise because the ethical views of society are such that society has a degree of “inequality aversion” with respect to the distribution of this health measure. In the latter instance the inequality aversion of society will be reflected in the way in which individual utility functions are aggregated into some measure of social welfare. In both cases social welfare (defined as some aggregate of individual welfare) will be sensitive to both the level and distribution of h .

In either case, comparison of the health measure will be influenced by the precise utility and/or social welfare function employed, since this will determine the relative importance attached to the average value of the health measure and its distribution. This can be problematic, since the ranking of any two populations may well be sensitive to the choice of specific utility/welfare function. This is where the issue of *dominance* enters the picture, since it refers to the situation where the ranking of two populations is not sensitive to the choice of specific utility/welfare function, providing such functions obey a limited number of properties, properties over which it is usually possible to obtain agreement.

Where dominance is not found, then analysts must rely upon comparisons of some measure of central tendency, usually the mean or the median. If distribution is also an issue they must rely upon specific utility/welfare function or, if the focus is solely upon distribution then specific inequality measures must be used running the risk that the ranking of populations may be sensitive to choice of function. In the case of health however, there may be a further complication. Some health measures are cardinal (for example life expectancy) and thus lend themselves to comparison via the mean and also via well-known inequality measures such as the Gini coefficient or coefficient of variation. In many cases however, the health measure is not cardinal but instead is ordinal and categorical, for example self-assessed health. In such cases analysts have essentially two choices: they can either transform their data from ordinal to cardinal, and then proceed using the cardinal approach referred to above. Alternatively, they can employ measures which are specifically designed to deal with ordinal data, bearing in mind however that the range of choice of such variables is quite restricted.

In this chapter we review the application of dominance methods and the measurement of inequality in health economics, for the case of both cardinal and ordinal data. We deal first of all with the case where the health measure is cardinal. Note that in the discussion which follows, we will be discussing what we could term “pure” health inequality i.e. inequality in health without reference to an individual’s socioeconomic resources. This distinguishes this review from the extensive literature on inequality in health outcomes with respect to income or other measures of resources (e.g. see Kakwani et al, 1997). We conclude with a brief discussion on statistical inference.

2. Dominance and Health Inequality with Cardinal Data

In analyzing issues of dominance and inequality in the case where health is measured cardinally, then the results and methods employed in the case of income inequality are available for use. It is probably easiest to deal with the case of inequality first. In what follows we assume we are making comparisons between two populations with respect to a measure of health h_i where it is assumed that higher values represent better health.

The primary dominance concept in the analysis of inequality is *Lorenz Dominance*. This involves comparison of the Lorenz curve for h_i for the two populations. The Lorenz curve orders individuals in increasing order of h_i and then plots, against the cumulative proportion of the population so ordered, the cumulative proportion of total health going to each proportion of the population. The graph corresponding to the 45 degree line represents complete equality – everyone has the same health. The closer the graph is to the 45 degree line, the more equal are the distributions. Thus if one distribution lies above (nearer to the 45 degree line) for all values of p then that distribution is said to

Lorenz dominate and would be ranked as more equal by all inequality measures obeying certain basic properties. These properties are anonymity (permutations of health among the population does not matter for overall inequality), population (the measure of inequality is independent of the size of the population), relativity (absolute levels of health do not matter for inequality measures) and transfer (inequality must fall if there is a transfer of a unit of health from a more to a less healthy person).

Where Lorenz dominance is found then the issue of inequality is essentially resolved. However it is frequently the case that dominance is not found, in which instance specific inequality measures must be used. There is a wide range of such measures. Among the most frequently used are the Gini coefficient, the coefficient of variation, the entropy family of measures and the Atkinson measure. In many instances it is possible to alter values of crucial parameters so that different weights can be put on inequalities in different parts of the distribution (e.g. the extended Gini of Donaldson and Weymark, 1980). A further additional property which may be desirable is that of decomposability i.e. where the population can be clearly partitioned e.g. by region, then the inequality index can be decomposed into inequalities within regions and inequalities between the regions. The only commonly used inequality index which can be perfectly decomposed (in the sense of there being no residual) is the Theil index (one of the entropy family).

Lorenz dominance is concerned with comparing health in two populations purely on the basis of inequality, without any reference to the *average* level of health. From a social welfare perspective, we might be willing to trade off greater inequality of health for a higher average level. To take an extreme example, suppose in population Q we have complete equality of health, whereas in population P we have a high degree of

inequality, yet the least healthy person in P has higher health than the average level in Q. Many would regard P as having superior health to Q even though Q Lorenz dominates P.

In these type of instances, stochastic dominance results can be applied. The degree of stochastic dominance will depend upon whether the data is cardinal or ordinal and also on the nature of the underlying utility function. Thus if we simply assume that individual utility is increasing in health, then dominance for population P over population Q holds if the cumulative distribution of health for population P first order stochastically dominates that for population Q.

If we assume that individual utility functions are not only increasing but also concave in the measure of health, then provided the health measure is cardinal, dominance may also be observed if the cumulative distribution of population P second order stochastically dominates that of population Q. In this case second order stochastic dominance is equivalent to what is known as Generalised Lorenz dominance (Shorrocks, 1983), where the Generalised Lorenz curve is simply the original Lorenz curve scaled up by the average level of health.

There is one further branch of dominance theory which is of relevance for comparison of some specific health measures between populations. In some cases we may be concerned if the value of a specific health measure lies above (or below) a critical threshold, while at the same time we are unconcerned should the value of the health measure be below (above) that threshold. This has clear parallels with the study of poverty and dominance results from the poverty literature can be applied in these cases (the seminal application of dominance theory in the poverty literature is Atkinson (1987)). One obvious area within health economics where such techniques could be

applied is obesity, with its focus on individuals whose body mass index lies above a critical threshold (see, for example, Madden, 2011). This approach is particularly useful when there may not be complete agreement over where the critical threshold should be drawn. A further example of an application of this technique in health economics is with regard to mental stress (Madden, 2009). Here mental stress is measured via a Likert scale derived from answers to the General Health Questionnaire (GHQ) and once again the threshold value of the scale which indicates mental stress is open to question. Stochastic dominance techniques are used here to show that regardless of where the threshold is drawn, there was a fall in mental stress in Ireland over the 1994 to 2000 period.

The analysis of mental stress in Ireland in Madden (2009) essentially interpreted the Likert scale derived from the GHQ as a cardinal measure of mental health. Strictly speaking this is not true as the underlying data used to construct the scale are of an ordinal categorical nature. Much health data, including the frequently encountered self-assessed health measures are of this nature and the application of dominance techniques and the calculation of inequality in these instances raise particular questions, to which we now turn.

3. Dominance and Inequality with Cardinal Data

While there are some health measures which are cardinal they tend to concentrate only on specific dimensions of health e.g. BMI. More general cardinal health measures are comparatively difficult to come across. Measures such as the SF-36 or Euroqol are available only for a limited range of countries. Probably the most frequently employed

measure of general health is self-assessed health (SAH). Individuals answer a question of the form: in general, how good would you say your health is? The possible answers are: very bad, bad, fair, good and very good (the exact wording can differ from survey to survey but it is generally of the above type). While this measure appears to give a good indicator of overall health (Idler and Benyamini, 1997) it is not cardinal, and with only five categories, it is not suited to the application of the standard inequality indices referred to above.

The breakthrough in analyzing inequality with such data came from Allison and Foster (2004). They show how standard measures of the spread of a distribution which use the mean as a reference point, such as the Gini, are inappropriate when dealing with categorical data. This is because the inequality ordering will not be independent of the (arbitrarily chosen) scale applied to the different categories. In this instance a more appropriate reference point is the median category and the cumulative proportions of the population in each category is the foundation of their analysis of inequality with categorical data. This is because while changes in the scale used will affect the width of the steps of the cumulative distribution, the height of the cumulative distribution is invariant to the choice of scale, thus providing the crucial property of scale independence.

Allison and Foster (2004) thus develop a partial ordering based on a median-preserving spread of the distribution (analogous to the partial ordering based on a mean preserving spread provided by say a Lorenz comparison). Thus suppose we have a measure of SAH with n different categories which can be clearly ordered $1, \dots, n$. Let m denote the median category and let P and Q denote two cumulative distributions of SAH with P_i and Q_i indicating the *cumulative* proportion of the population in category i , in

each distribution, where $i=1, \dots, n$. For the case where both P and Q have identical median states m then P has less inequality than Q if for all categories $j < m, P_j \leq Q_j$ and for all $j \geq m, P_j \geq Q_j$. What this is effectively saying is that distribution Q could be obtained from distribution P via a sequence of median-preserving spreads.

Allison and Foster also deal with dominance when the focus is on the level of the health measure. In this case distribution P will dominate distribution Q if the cumulative frequency at each point on the ordinal scale (as we go from lower to higher) is always higher in Q than in P . This is equivalent to the first order stochastic dominance condition referred to above. For a recent example of application of this approach to a comparison of SAH between different social classes, see Dias (2009). It is important to note that when data are ordinal then second order stochastic dominance is not defined, since it requires that the health measure h can be summed in a meaningful way.

Of course, the Allison-Foster measure shares with Lorenz dominance the property that it only provides a partial ordering and there may be instances when the above conditions do not hold and it is not possible to rank different distributions of categorical data. Abul Naga and Yalcin (2008) address this issue and build upon the Allison-Foster approach in presenting a parametric family of inequality indices for qualitative data. Like its cardinal data counterparts such as the Gini coefficient or coefficient of variation, it will always provide a ranking, but it lacks the generality of the dominance approach. Subsequent to the Abul Naga-Yalcin paper, Lazar and Silber (2011) have provided an alternative index for ordinal data building upon work by Reardon (2009) in the area of ordinal segregation. Abul Naga and Yalcin (2010) have also extended their work in a

very useful direction by providing an index which can be used to make comparisons when the two distributions in question do not have the same median category.

The Allinson-Foster, Abul Naga-Yalcin and Lazar-Silber contributions show that real progress has been made towards measuring inequality in the case of ordinal data. However, at this stage in the literature there is still only a limited number of indices specifically designed for ordinal data, so, unlike the case with cardinal data, the analyst has less opportunity to check the sensitivity of results to alternative indices. In this instance there is another approach which can be taken. It is possible to transform ordinal data to cardinal data, and then apply the cardinal indices referred to above.

Much of the literature in this area developed in the context of measuring health inequality related to socio-economic resources and a very useful summary is available in Jones and van Doorslaer (2003). Their favoured approach is to use interval regression to obtain a mapping from the empirical distribution function of what is regarded as a valid index of health (such as the McMaster Health Utility Index (HUI)) to SAH. By mapping from the cumulative frequencies of SAH categories into an index of health such as the McMaster HUI it is possible to obtain upper and lower limits of the intervals for the SAH categories. These can then be used in an interval regression to obtain a predicted value of the index for all individuals. Comparisons which they carry out for measures of SAH in Canada suggest that this approach to cardinalisation outperforms other approaches. Research by Van Doorslaer and Koolman (2004) and Van Ourti et al (2006) indicates that the values of the health index obtained are not very sensitive to the cut-off points chosen (see also Lecluyse and Cleemput, 2006, and Lauridsten et al, 2004). Hence it may be

regarded as acceptable to use cut-off points from the Canadian HUI to calculate a cardinal index of health for other countries.

A key question then is, how do the results obtained from such an approach compare with those from an index specifically designed to deal with ordinal data? Madden (2010) carried out such an exercise, calculating ordinal inequality indices using the Abul-Naga approach and also cardinal indices using generalized entropy measures and applying them to Irish data for the years 2003-2006. In terms of the ranking of the different years there was very little correlation between the ordinal and cardinal indices. This is a specific result obtained with a specific dataset but it underlines that the choice between the application of an ordinal index versus transforming data into cardinal format and then using a cardinal index may not be trivial.

4. Statistical Inference

Sections 2 and 3 outlined approaches to testing for dominance and measuring inequality, using both cardinal and ordinal data. Should dominance be found then of course it is necessary to check if such a finding is statistically significant. Similarly, we may wish to calculate the standard errors associated with any particular index of inequality which we calculate.

Dealing first with dominance in the case of cardinal data, in the case of inequality alone this issue boils down to checking for statistically significant differences between the ordinates of the Lorenz curves. Suppose that L_i is the i^{th} Lorenz ordinate ($(i = 1, 2, \dots, k)$), where the k^{th} ordinate is equal to one. Then, as shown in Beach and Davidson (1983), given estimated Lorenz ordinates from two populations P and Q with

sample sizes N_P and N_Q respectively, we have $k-1$ pairwise tests of sample Lorenz ordinates:

$$T_i = \frac{\hat{L}_i^P - \hat{L}_i^Q}{\sqrt{\frac{\hat{V}_i^P}{N_P} + \frac{\hat{V}_i^Q}{N_Q}}}, \quad i = 1, 2, \dots, k-1$$

In large samples, T_i is asymptotically normally distributed. Bishop, Formby and Smith (1991) suggest the following criteria when testing for Lorenz dominance: if there is at least one positive significant difference and no negative significant differences between Lorenz ordinates then dominance holds. Two distributions are ranked as equivalent if there are no significant differences, while the curves cross if the difference in at least one set of ordinates is positive and significant while at least one other set is negative and significant.

In the case of first and second order stochastic dominance for cardinal data, then Kolmogorov-Smirnov tests can be applied, as described in Davidson and Duclos (2000). Such tests can also be applied to ordinal data for first order stochastic dominance

If Lorenz dominance is not found then individual inequality indices must be calculated and the appropriate standard error obtained. Obtaining analytic expressions for standard errors in the case of many inequality indices is far from easy as the expressions may be highly non-linear and while asymptotic results may exist, robust small sample results are more difficult to obtain (an exception to this is Biewen and Jenkins, 2006). Given this problem, the bootstrap approach may be preferable, as evidence suggests that bootstrap tests perform reasonably well in these situations (see Mills and Zandvikli, 1997 and Biewen, 2002).

In the case of the ordinal inequality index developed by Abul Naga and Yalcin to date there has been no progress in terms of statistical inference for this index.

5. Conclusion

This chapter has summarised some of the main results with respect to dominance and inequality in the case of health data. It was seen that a crucial distinction must be made between cardinal and ordinal health measures. In general the literature for cardinal health measures is more developed, in terms of dominance, indices and statistical inference. For the case of ordinal health measures, which are arguably more widely employed, dominance results are generally less applicable, there are fewer inequality indices and statistical inference is less well developed.

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