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Monetary Penalties and Politician Preferences**

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# Caps on Political Contributions, Monetary Penalties and Politician Preferences

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## **Abstract**

With politician preferences over policy outcomes, the effect of a contribution cap with monetary penalties for exceeding the cap is starkly different from the case with an indifferent politician. In contrast to Kaplan and Wettstein (AER, 2006) and Gale and Che (AER, 2006), a cap is never neutral on the expected cost of contributions nor on the policy outcome. Furthermore more restrictive caps can lead to increased aggregate contributions. When the penalty for exceeding the cap is small enough that it is impossible to suppress all contributions, the influence of money on policy is minimized with a binding but non-zero cap and maximized with no cap.

Keywords: All-pay auction, campaign finance reform, soft money, explicit ceiling, BCRA.  
JEL: D72, C72

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A basic premise of representative democracy is that people elect representatives who either share their policy preferences or are willing to act as if they do in order to be reelected. In either case it is likely that politicians have preferences over policy alternatives. However it is often argued that the need to raise money to run a political campaign may weaken the connection between politicians' policy preferences and the actions of politicians, undermining this fundamental premise of representative democracy.<sup>1</sup> The concern is that politicians' need to raise funds may lead to large contributors having undue policy influence.<sup>2</sup> In order to achieve a "Reduction of Special Interest Influence" (Title I of Public Law 107-155, 107<sup>th</sup> Congress) the Bipartisan Campaign Reform Act of 2002 (commonly known as the McCain-Feingold Bill) re-regulates campaign contributions. It sets clear limits on individuals' contributions to candidates and to political action committees.<sup>3</sup>

On the other hand, it is also suggested that contributions themselves may help politicians learn about agents' valuations of alternative policies. In this view it is important not to restrict contributions because they contain information about the social changes desired by the people.<sup>4</sup> This paper does not take sides in this debate, but rather provides a positive analysis of the effects of a contribution cap where the incumbent politician has a policy preference and his policy choice may be swayed by political contributions. The previous literature examining the effect of contribution

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<sup>1</sup>According to Senator Russ Feingold (D-WI) "because it costs so much to run for office, interests with big money to contribute to candidates or spend on ad campaigns are able to get special access in Congress." Senator John McCain (R-AZ) argues: "Americans believe that political representation is measured on a sliding scale. The more you give the more effectively you can petition your government." (Quoted on the Senators' web sites, March 2008). Politicians are forced to raise ever-increasing funds. In 2008 the average cost of a successful campaign for the House of Representatives was \$1.3 million, which represents a real increase of 53% in a decade. Over the same period the average cost of a winning Senate campaign increased by 21% in real terms to \$6.5 million. For summary statistics see the web sites of the Campaign Finance Institute and the Center for Responsive Politics, [www.cfinst.org](http://www.cfinst.org) and [www.opensecrets.org](http://www.opensecrets.org) respectively.

<sup>2</sup>It is well documented that institutional contributors appear to be acting as rational investors (see for instance Kroszner and Stratmann (1998) and Snyder (1990)) and special interest groups lobby members with positions of power in congressional committees more heavily, see Ansolabehere *et al.* (2003) for a literature survey.

<sup>3</sup>The act limits an individual's contributions to a candidate to a maximum of \$2,300 per election and to a political action committee to a maximum of \$5,000 with built-in increases for inflation. See <http://www.fec.gov/pages/brochures/contriblimits.shtml> for details. For state-level offices individual states are in charge of their own campaign finance regulations. All states except for Illinois, New Mexico, Oregon, Utah and Virginia have contribution limits. Details on various state level contribution limits are provided by the National Conference of State Legislatures, [www.ncsl.org](http://www.ncsl.org). A number of other countries also have contribution limits. Examples include France, India, Israel, Italy, Japan, Mexico, Russia, Spain, Taiwan and Turkey. See [www.aceproject.org](http://www.aceproject.org).

<sup>4</sup>The landmark Supreme Court ruling on *Buckley v. Veleo* argues that the Constitution affords the broadest protection to political expression in order to assure the "unfettered interchange of ideas for the bringing about of political and social changes desired by the people." Opponents of limits on political contributions frequently point to Chief Justice Berger's dissenting opinion extending this argument on campaign spending to political contributions.

caps on lobbyists' ability to buy political favors analyzes the implications of a contribution cap under the assumption that the politician is indifferent between the policy positions of the lobbyists.

A seminal paper by Che and Gale (1998) shows that a rigid cap on political contributions can have the unexpected consequence of increasing contributions. A restrictive cap can level the playing field, resulting in more intense competition between lobbyists. Kaplan and Wettstein (2006), henceforth KW, analyzes the effect of a cap in a more realistic setup where it is possible to give more than the specified limit albeit with a risk of being caught. If the contribution cap is non-rigid, not only does the Che and Gale (1998) result disappear, but a cap is completely neutral on lobbying costs and on the outcome of the competition. Exceeding the cap comes at a cost, resulting in a kink in the cost of contributions. But the cap does not alter the relative strengths of the lobbyists, so it does not change the intensity of their competition. Gale and Che (2006), henceforth GC, further refines KW and shows that the original Che and Gale (1998) result can still hold if the punishment for violating the contribution cap is non-monetary (such as jail time) and lobbyists initially have asymmetric costs. But if the punishment is monetary, contribution caps are completely neutral on lobbying costs whether the cost functions are asymmetric or not (GC Proposition 1, Corollary 1).

In the United States monetary penalties for violation of contribution limits are the norm. While Sections 312-315 of Title III, Public Law 107-155, 107<sup>th</sup> Congress allow criminal prosecution of violators, in almost all cases violations are handled as a purely administrative matter by the Federal Elections Commission and only the civil penalties under section 315 are used. From 2003 through 2008 the FEC imposed penalties in 600 cases imposing fines totaling \$18,641,360.<sup>5</sup> Together with KW/GC this suggests that under the current enforcement regime the policy discussion about the effect of contribution limits on lobbyists' ability to buy political favors is a red herring: Unless criminal penalties are enforced contribution limits have no effect on lobbying costs or the policies that politicians enact.

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<sup>5</sup>See [www.fec.gov](http://www.fec.gov) for details. In January 2009 the Criminal Division of the US Department of Justice put in a request to change the practice of administrative penalties being imposed by the FEC without Justice Department input. The Justice Department asks that the FEC review the memorandum of understanding between the departments with a view to permitting the inclusion of a US Attorney in the FEC's decision whether or not to forward cases for possible criminal prosecution.

In this paper we revisit the discussion of monetary punishments for violating a contribution cap by introducing politician policy preferences into the KW/GC framework.<sup>6</sup> We show that with politician preferences contribution caps are never neutral on expected lobbying costs, even with purely monetary punishments. Moreover we recover the original Che and Gale (1998) result that contribution limits may intensify competition between lobbyists resulting in increased expected levels of contributions.

Even when the cost functions of the lobbyist are identical, we show that a non-rigid cap does not have a symmetric effect on the lobbyists when the politician has a policy preference. The lobbyist with the disfavored policy alternative must contribute more than his rival to overcome the preference of the politician. Hence starting from a non-binding cap making the cap more restrictive imposes a cost on the disfavored lobbyist before it imposes a cost on the lobbyist with the favored policy position. This tilts the playing field in favor of the lobbyist with the preferred policy position which can induce more aggressive competition leading to higher contributions and costs.

The introduction of politician policy preferences allows us to broaden the discussion of the effect of contribution caps beyond the expected level of contributions. We examine an additional metric for the degree of influence of political contributions on policy: The probability that the politician does not make the policy decision that he would have made in the absence of political contributions. The choice of measure matters. With policy preferences a cap may lead to increased aggregate contributions while at the same time making it more likely that the politician enacts his preferred policy, reducing the influence of lobbying effort. We show that the influence of political contributions on policy is at its minimum with a binding but strictly positive cap on contributions for any set of parameter values where it is not possible to completely suppress all political contributions. In addition, for all sets of parameter values we show that any binding contribution cap will reduce the influence of political contributions on policy compared to no contribution limits. In addition we show that contribution caps may redistribute political contributions from Senators and politicians from large or urban districts to Representatives and politicians from smaller or rural districts.

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<sup>6</sup>There is extensive empirical evidence indicating that the policy position of the politician is an important determinant of politician behavior. Of the 36 empirical papers which study ideology or party affiliation surveyed in Ansolabehere *et al* (2003), all but one find policy position significant for predicting congressional roll-call votes.

## I. Model

Two risk-neutral lobbyists compete for a political prize. The prize will be arising due to a policy choice of a politician who holds a political post. The prize may be a vote on impending legislation but may also be more subtle, such as attaching a rider to an upcoming bill creating a regulatory loophole, or pushing a particular wording in committee. The value of the political prize to lobbyist 1 is denoted by  $v_1$ , and the value of the prize to lobbyist 2 is  $v_2$ ,  $v_1 > v_2 > 0$ . The lobbyists make simultaneous contributions,  $x_1$  and  $x_2$ , to the politician in power. The contributions are not returned to the lobbyist whose efforts fail. If lobbyist 1 wins the prize, his payoff is  $v_1 - c(x_1)$  where  $c(x_j)$  is the cost of contributing  $x_j$ . If the rival wins, lobbyist 1's payoff is  $-c(x_1)$ . Lobbyist 2 has an identical cost function and his payoffs are constructed in the same manner. Since the contributions are sunk both for the winner and the loser, this political lobbying game is an all-pay auction.<sup>7</sup>

The politician has a preference over the policy alternatives supported by the two lobbyists. The politician's preference may be ideologically based or it may be induced from the preference of the constituents who will be voting in the future. The interest groups lobby the politician and the politician awards the political prize based on the contributions and his preference. The intensity of the preference for the policy position of lobbyist 2 is put into monetary terms, denoted by  $\gamma \in (-\infty, \infty)$ . For example  $|\gamma|$  could represent the expected future campaign costs required to offset the effect of taking a policy position that is unpopular in the politician's district. If the politician favors lobbyist 2's position,  $\gamma > 0$ . If the politician favors lobbyist 1's position,  $\gamma < 0$ . The politician awards the prize to lobbyist 1 if  $x_1 > x_2 + \gamma$ . In case of a tie, each contestant has an even chance of winning the prize.<sup>8</sup> The rules of the game, the valuations of the lobbyists and the preference of the politician are common knowledge.

Simple backward induction in the one-shot game that is analyzed here would have the politician taking his preferred action regardless of contributions since all contributions are sunk. Hence there would be no contributions. Thus implicitly we are assuming that this one-shot game is

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<sup>7</sup>The all-pay auction without a cap has been analyzed by Hillman and Riley (1989) and Baye *et al.* (1993, 1996). The results in Siegel (2009) are applicable with or without a cap. See Yildirim (2005) for a contest where players have the flexibility to add to their previous efforts and see Kaplan, Luski and Wettstein (2002) for a model where the size of the reward is a function of the bid.

<sup>8</sup>Konrad (2002) analyzes an all-pay auction with additive preferential treatment, equal bidder valuations and no cap. We extend Konrad (2002) to allow bidders with different valuations and introduce a contribution cap.

embedded in a repeated setting so that the politician has an incentive to reward high contributions in order to keep them coming in the future. However, as long as contributions, preferences and actions are common knowledge among lobbyists, the same lobbyists do not necessarily need to be involved in repeated contests.

Both lobbyists have an identical constant marginal cost of raising funds, normalized to one. If the lobbyist exceeds the cap, in expectation he pays a monetary fee of  $a(x - m)$  where  $m \in [0, \infty)$  is the contribution cap and  $0 < a < \infty$ .<sup>9</sup> Hence the expected cost of making a political contribution  $x_i$  is given by the cost function:

$$c(x_i) = \begin{cases} x_i & \text{if } x_i \leq m \\ x_i + a(x_i - m) & \text{if } x_i > m \end{cases} \quad (1)$$

As in KW and GC the cost function is continuous in  $x$ . Section 315 of Title III, Public Law 107-155 specifies that violations will incur a fine which is “not less than 300 percent of the amount involved in the violation and is not more than the greater of \$50,000 or 1000 percent of the amount involved in the violation.” Thus under the current legislation if a violation is caught with certainty the relevant  $a$  is between 3 and 10. If there is a chance that the violation will not be caught the relevant  $a$  is correspondingly lower.

It will prove useful to define  $\bar{x}_i$  as the maximum contribution the lobbyist would ever be willing to make. It is the level of contribution where  $c(\bar{x}_i) = v_i$ :

$$\bar{x}_i = \begin{cases} v_i & \text{if } v_i \leq m \\ \frac{v_i + am}{1 + a} & \text{if } v_i > m \end{cases} \quad (2)$$

It is possible to write the equilibrium strategies more concisely if we define  $f$  as the lobbyist whose policy is favored by the politician and  $u$  as the lobbyist with the unfavored policy. So if  $\gamma > 0$ ,  $f=2$  and  $u=1$ , while if  $\gamma < 0$ ,  $f=1$  and  $u=2$ . It will also prove useful to define the “Advantaged” lobbyist,  $A$ , as the lobbyist with the highest reach:  $A$  is the lobbyist who can have a positive payoff by outbidding his rival even if his rival is bidding her maximum willingness to contribute. So lobbyist  $u$  is the

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<sup>9</sup>The effect of a rigid cap (where the lobbyists cannot exceed the specified contribution limit,  $a \rightarrow \infty$ ) with politician preferences is analyzed in Pastine and Pastine (2008).

advantaged lobbyist if  $\bar{x}_u \geq \bar{x}_f + |\gamma|$  and lobbyist  $f$  is the “Disadvantaged” lobbyist  $D$ . Similarly, lobbyist  $f$  is the advantaged lobbyist if  $\bar{x}_f > \bar{x}_u - |\gamma|$ ,  $f=A$  and  $u=D$ .

## II. Equilibrium

If the preference of the politician is too strong,  $v_u \leq |\gamma|$  even if the favored lobbyist makes no contribution it would not be worthwhile for the unfavored lobbyist to compete. Hence when  $\gamma \leq -v_2$  ( $u=2$ ) or  $\gamma \geq v_1$  ( $u=1$ ), equilibrium is only in pure-strategies where neither lobbyist contributes.

If the preference of the politician is not too strong,  $\gamma \in (-v_2, 0) \cup (0, v_1)$  it may be possible to suppress all competition with a sufficiently restrictive cap provided that the penalty for exceeding the cap is high enough. The lobbyist with the unfavored policy position must contribute at least  $|\gamma|$  in order to have a non-zero chance of winning. Competition is completely suppressed if the cost of contributing  $|\gamma|$  meets or exceeds the value of winning the prize for the lobbyist with the unfavored policy position:  $|\gamma| + a(|\gamma| - m) \geq v_u$ . Solving for the highest  $m$  that can suppress all contributions:

$$m^* = \frac{(1+a)|\gamma| - v_u}{a} \quad (3)$$

$m^*$  is positive when the punishment for violating the cap is large enough,  $a > (v_u - |\gamma|) / |\gamma|$ . If  $m^* \geq 0$  then for all  $m \leq m^*$  competition is completely suppressed. In this case neither lobbyist contributes and the political prize is allocated to the lobbyist with the favored policy alternative. If  $m^* < 0$ , competition cannot be completely suppressed no matter how restrictive the cap may be. There will be competition for the political prize through political contributions even if all contributions are banned. Proposition 1 below gives the equilibrium strategies for all  $\gamma \in (-v_2, 0) \cup (0, v_1)$  and  $m > \min(0, m^*)$ . Define the interval  $[b, c]$  as the empty set whenever  $b \geq c$ , and similarly define open intervals.

**Proposition 1:** *For all  $\gamma \in (-v_2, 0) \cup (0, v_1)$  and  $m > m^*$  equilibrium is only in mixed-strategies and it is characterized by unique probability density functions  $F_f(x)$  and  $F_u(x)$  for the favored lobbyist’s and the unfavored lobbyist’s political contributions, respectively. These distributions are continuous in  $m$ .*



(i) When  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$  or when  $\gamma \in \left( \frac{v_1 - v_2}{1+a}, v_1 - v_2 \right]$  and  $m \leq \frac{(1+a)(v_2 + \gamma) - v_1}{a}$ , then  $f=A$  and  $u=D$ . The unique equilibrium distribution functions are given by:

$$F_u(x_u) = \begin{cases} \frac{v_f - c(\bar{x}_u - |\gamma|)}{v_f} & \text{for } x_u \in [0, |\gamma|] \\ \frac{v_f - c(\bar{x}_u - |\gamma|) - |\gamma| + x_u}{v_f} & \text{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_u)] \\ \frac{v_f - c(\bar{x}_u - |\gamma|) - am + (1+a)(x_u - |\gamma|)}{v_f} & \text{for } x_u \in (m + |\gamma|, \bar{x}_u] \\ 1 & \text{for } x_u \in (\bar{x}_u, \infty) \end{cases}$$

$$F_f(x_f) = \begin{cases} \frac{|\gamma| + x_f}{v_u} & \text{for } x_f \in [0, \min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|)] \\ \frac{(1+a)|\gamma| - am + (1+a)x_f}{v_u} & \text{for } x_f \in [\max(0, m - |\gamma|), \bar{x}_u - |\gamma|] \\ 1 & \text{for } x_f \in [\bar{x}_u - |\gamma|, \infty) \end{cases}$$

(ii) When  $\gamma \in \left( 0, \frac{v_1 - v_2}{1+a} \right]$ , or when  $\gamma \in \left( \frac{v_1 - v_2}{1+a}, v_1 - v_2 \right]$  and  $m > \frac{(1+a)(v_2 + \gamma) - v_1}{a}$ , then  $f=D$  and  $u=A$ . The unique equilibrium distribution functions are given by:

$$F_u(x_u) = \begin{cases} 0 & \text{for } x_u \in [0, |\gamma|] \\ \frac{-|\gamma| + x_u}{v_f} & \text{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_f + |\gamma|)] \\ \frac{-(1+a)|\gamma| - am + (1+a)x_u}{v_f} & \text{for } x_u \in (m + |\gamma|, \bar{x}_f + |\gamma|] \\ 1 & \text{for } x_u \in (\bar{x}_f + |\gamma|, \infty) \end{cases}$$

$$F_f(x_f) = \begin{cases} \frac{v_u - c(\bar{x}_f + |\gamma|) + |\gamma| + x_f}{v_u} & \text{for } x_f \in [0, \min(\max(0, m - |\gamma|), \bar{x}_f)] \\ \frac{v_u - c(\bar{x}_f + |\gamma|) - am + (1+a)(x_f + |\gamma|)}{v_u} & \text{for } x_f \in [\max(0, m - |\gamma|), \bar{x}_f] \\ 1 & \text{for } x_f \in [\bar{x}_f, \infty) \end{cases}$$

**Proof:** Appendix A.

Proposition 1 completely describes the equilibrium for any set of parameter values where there is competition between lobbyists and  $\gamma \neq 0$ . We can treat the no-cap case as  $m \rightarrow \infty$ . Equilibrium of this contribution game does not exist in pure strategies. The best response to a bid  $b'$  of the favored bidder is either to outbid him by  $|\gamma|$  or to drop out of the race. In either case the favored bidder's choice of  $b'$  would not be optimal.

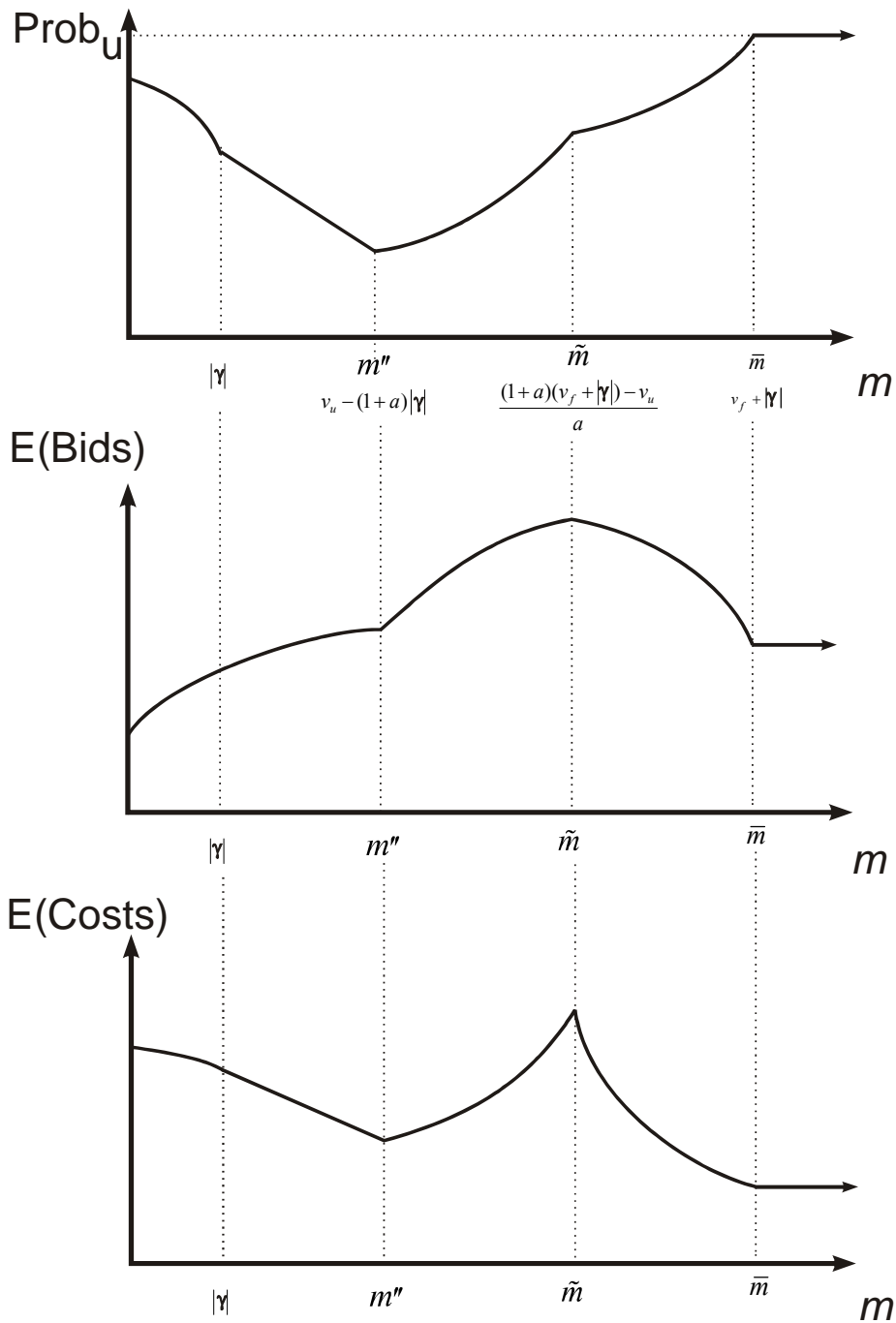
Since equilibrium is in mixed strategies, the unfavored lobbyist may end up contributing more than the favored lobbyist but not by enough to overcome the politician's preference  $|\gamma|$ . Consequently, the empirical evidence on the effect of money on legislative action may appear to be weak. Indeed in their survey Ansolabehere *et al.* (2003) find that the effect of PAC contributions on roll-call votes is mixed.<sup>10</sup> Furthermore given that the preference of the politician would vary over policy issues, the model is consistent with the fact that the evidence appears to be strong in some policy areas but not in others.

From the equilibrium distribution functions it is straightforward to examine the probability that the lobbyist with the unfavored policy position wins,  $Prob_u$ , the expected total contributions,  $E(Bids)$ , and the expected total lobbying costs  $E(Costs)$ . To illustrate the forces at work, Figure 1 presents  $Prob_u$ ,  $E(Bids)$  and  $E(Costs)$  as a function of the cap. Inflection points and kinks in the graphs result from the existence or non-existence of kinks in the distribution functions. The figure is drawn for  $\gamma \in \left( \frac{v_1 - v_2}{1 + a}, v_1 - v_2 \right]$  and  $a \in \left( \frac{v_1 - 2v_2}{v_2}, \frac{v_1 - v_2}{v_2} \right)$ . These ranges for  $\gamma$  and  $a$  are chosen in order to present graphs with the richest results which permits a discussion of the intuition for all the critical values of  $m$ .

Given the degree of policy preference of the politician, there are five critical values for the level of cap that determine the existence or nonexistence of kinks in the lobbyists' equilibrium

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<sup>10</sup>They survey 34 empirical papers and find that evidence on the effect of PAC contributions on roll-call votes is strong in some policy areas but not in others. For instance, on issues relating to trade there is weak evidence of the effect of PAC contributions on votes, but on issues relating to labor the evidence is very strong.



**Figure 1:** Equilibrium results for  $\gamma \in \left( \frac{v_1 - v_2}{1+a}, v_1 - v_2 \right]$  and  $a \in \left( \frac{v_1 - 2v_2}{v_2}, \frac{v_1 - v_2}{v_2} \right)$ . Therefore  $f=2$ ,  $u=1$ . Proposition 1 (i) applies when  $m \leq \tilde{m}$ . Proposition 1 (ii) applies when  $m > \tilde{m}$ .

distribution functions:  $m^*, \bar{m}, m'', \tilde{m}, |\gamma|$ . These critical values are derived and discussed below. The equilibrium implications are presented in the next section. But first it will be useful to define some terminology. A “binding cap” is a cap which is lower than the maximum of the upper bounds of the

no-cap equilibrium bid supports of the lobbyists. A “*more restrictive cap*” refers to a smaller  $m$  when the cap is binding.

$m^*$  is the level of the cap that completely suppresses all competition, which is given by (3). In the range of  $\gamma$  and  $a$  for which Figure 1 is presented,  $v_u > (1+a)|\gamma|$  and hence  $m^* < 0$ . So even when all contributions are banned there is still competition over the political prize and therefore  $Prob_u > 0$  at  $m=0$ .

$\bar{m}$  denotes the level of a barely binding cap. It is equal to the maximum of the suprema of the no-cap equilibrium contribution supports of the two lobbyists.

$$\bar{m} = \begin{cases} v_u & \text{when } \gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1) \\ v_f + |\gamma| & \text{when } \gamma \in (0, v_1 - v_2] \end{cases} \quad (4)$$

The cap is not binding when  $m > \bar{m}$ . An increase in  $m$  above  $\bar{m}$  has no effect on the equilibrium of the game and hence  $Prob_u$ ,  $E(Bids)$  and  $E(Costs)$  remain constant.

$m''$  is the third critical value. When  $m > m''$  the lobbyist with the favored policy position never exceeds the cap in equilibrium. From Proposition 1 and Lemma 5 in Appendix A, the upper bound of the equilibrium distribution function of the favored lobbyist is:

$$x_f^{\text{sup}} = \begin{cases} \bar{x}_u - |\gamma| & \text{when Proposition 1(i) applies} \\ \bar{x}_f & \text{when Proposition 1(ii) applies} \end{cases} \quad (5)$$

Solve for the lowest  $m$  where  $x_f^{\text{sup}} \leq m$  is satisfied using equation (2) and Proposition 1:

$$m'' = \begin{cases} v_u - (1+a)|\gamma| & \text{when } \gamma \in (-v_2, 0) \cup \left( \frac{v_1 - v_2}{1+a}, v_1 \right) \\ v_f & \text{when } \gamma \in \left( 0, \frac{v_1 - v_2}{1+a} \right] \end{cases} \quad (6)$$

The lobbyist with the favored policy alternative has a positive probability of exceeding the cap when  $m < m''$  but he never exceeds the cap if  $m \geq m''$ . This results in a kink in the equilibrium distribution function of the unfavored lobbyist.<sup>11</sup>

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<sup>11</sup>In Proposition 1(i)  $\min(m + |\gamma|, \bar{x}_u) = m + |\gamma|$  if  $m < m'' = v_u - (1+a)|\gamma|$ . In Proposition 1(ii)  $\min(m + |\gamma|, \bar{x}_f + |\gamma|) = m + |\gamma|$  if  $m < m'' = v_2$ .

When  $m \in (m'', \bar{m})$ , it is only the lobbyist with the unfavored policy position who has a positive probability of exceeding the cap. In this range of  $m$ , a less restrictive cap just decreases the cost of the unfavored lobbyist and hence it makes him relatively more aggressive which leads to an increase in the probability that the unfavored lobbyist wins, as can be seen in Figure 1. When  $m < m''$ , in equilibrium both lobbyists have a positive probability of exceeding the cap. Since the unfavored lobbyist must exceed his rival's contribution by  $|\gamma|$  in order to win, he pays a higher expected cost of exceeding the cap. If the cap is relaxed both lobbyists pay less in expected cost of exceeding the cap. But the reduction is a larger portion of the total contribution costs for the favored lobbyist. Hence, when  $m < m''$ , an increase in  $m$  leads to a decrease in the probability that the unfavored lobbyist wins.

In Figure 1,  $m''$  is also a local minimum for the expected total costs. When  $m < m''$ , a less restrictive cap leads to a reduction in the expected total cost, since in expectation the lobbyists pay less extra cost for exceeding the cap. When  $m \geq m''$ , a less restrictive cap gives a favorable tilt to the playing field for the unfavored lobbyist and this induces more aggressive bidding by the unfavored lobbyist hence the expected aggregate costs increase.<sup>12</sup>

$\tilde{m}$  is the level of cap where the playing field is leveled,  $\bar{x}_u = \bar{x}_f + |\gamma|$ . This critical value only arises when  $\gamma \in ((v_1 - v_2) / (1 + a), v_1 - v_2]$ . For all other values of  $\gamma$ , as the level of the cap changes, the identity of the advantaged lobbyist  $A$  remains the same.

$$\tilde{m} = \frac{(1 + a)(v_2 + |\gamma|) - v_1}{a} \quad (7)$$

See Lemma 7 in Appendix A for the derivation of  $\tilde{m}$ . At  $\tilde{m}$  the advantage in the game switches from one lobbyist to the other. In this range  $\gamma \in ((v_1 - v_2) / (1 + a), v_1 - v_2]$  when  $m \leq \tilde{m}$ ,  $f=A$  and Proposition 1(i) describes the equilibrium distribution functions. The policy preference and the restrictive nature of the contribution limit legislation yield a playing field that is tilted in favor of the low-valuation lobbyist whose policy position is favored. When  $m > \tilde{m}$ ,  $u=A$  and Proposition 1(ii) applies. Due to his higher valuation of the prize, the unfavored lobbyist can offset the disadvantage arising from the

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<sup>12</sup>Note that when the penalty for exceeding the cap is sufficiently high  $a > (v_u - |\gamma|) / |\gamma|$ ,  $m''$  is negative and  $m^*$  is positive. This implies that the favored lobbyist never exceeds the cap for these parameter values and it is only the unfavored lobbyist who is directly affected by a relaxation in the contribution cap.  $\text{Prob}_u$  then does not have a U shape as depicted in Figure 1. It is zero for all  $m \leq m^*$  and it is an increasing function of  $m$  for all  $m > m^*$ .

preference of the politician in an environment where the contribution cap is not too restrictive. At  $\tilde{m}$ , the leveled playing field induces fierce competition and both lobbyists are most aggressive in their contributions;  $E(Bids)$  are maximized.

$m = |\gamma|$  also yields a kink in the graphs. When  $m < |\gamma|$  the unfavored bidder exceeds the cap for all strictly positive bids in the support of his equilibrium strategy. However, when  $m > |\gamma|$  this is not the case: The unfavored lobbyist has a kink in his cost function at  $m$ . This causes a kink in the equilibrium distribution function of the favored lobbyist:  $\max(0, m - |\gamma|) = 0$  when  $m > |\gamma|$ . In Figure 1 this translates into a kink in the graphs at  $|\gamma|$ .

### III. Implications

In this section we present three main implications of the equilibrium which provide insight into the effects of a contribution cap with monetary sanctions when the politician has a preference over policy outcomes.

**Result 1:** *A binding cap is never neutral on expected costs for  $\gamma \neq 0$ .*

**Proof:** Appendix B.

In KW/GC a non-rigid cap with a monetary punishment is always neutral on expected costs. The cap introduces a kink in the cost function of each lobbyist, but the effect of the kink is the same for both of them. So rather than thinking of the lobbyists as choosing contributions it is possible to simply think of them as choosing cost. The cap does not change their relative strengths in this competition. Hence it has no effect on the competition in costs if the politician has no policy preference.

However, when the politician has a policy preference, a contribution cap no longer effects the two lobbyists in a symmetric fashion. The lobbyist with the unfavored policy must outbid the favored lobbyist in order to overcome the policy preference. This means that any change in the level of the cap will have different effects on the two lobbyists. In equilibrium the unfavored lobbyist exceeds the cap for a greater portion of the range of his equilibrium strategy. Hence any change in the level

of  $m$  alters the lobbyists' relative strength in the competition and therefore it changes the intensity of the competition between them.

**Result 2:** *When  $\gamma \in (0, v_1 - v_2]$  there always exists a range of  $m$  where expected total costs and expected total contributions strictly increase with a more restrictive contribution cap. This range is:*

- $m \in [\max(m'', |\gamma|), \bar{m}]$  for  $\gamma \in \left(0, \frac{v_1 - v_2}{1 + a}\right]$
- $m \in (\max(\tilde{m}, |\gamma|), \bar{m})$  for  $\gamma \in \left(\frac{v_1 - v_2}{1 + a}, v_1 - v_2\right]$

**Proof:** Appendix B.

A more restrictive cap may have the perverse effect of increasing aggregate contributions if the politician mildly prefers the policy position of the low-valuation lobbyist and the cap is not too restrictive to begin with. Hence we retrieve the original result of Che and Gale (1998) without the use of non-monetary punishments. For the range of parameter values in Result 2 the politician prefers the policy position of the low-valuation lobbyist, however the preference is not strong enough to overcome bidder 1's advantage arising from his higher valuation. Additionally for the range of  $m$  given in Result 2,  $m > m''$  so only the unfavored lobbyist has a positive probability of exceeding the cap in equilibrium. Therefore decreasing  $m$  directly affects the unfavored lobbyist but does not directly affect the costs of the favored lobbyist. Since the unfavored lobbyist has the advantage in the competition this results in a more level playing field and triggers more aggressive bidding by the favored lobbyist resulting in higher aggregate contributions and cost.

The theory predicts that effect of the cap on aggregate contributions depends on the politician's preference and the initial level of the cap. Indeed empirically the effect of a cap does appear to depend on the environment. In gubernatorial elections from 1978 to 1997 Gross *et al.* (2002) finds that more restrictive contribution limits result in increased incumbent spending.<sup>13</sup> However the opposite seems to be the case for state legislative offices. Hogan (2000) finds that the incumbent spending goes down with stricter limits using data on state legislative candidates in mid-

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<sup>13</sup>Campaign expenditures closely track total campaign contributions since only one percent of total expenditures are self financed, Herrnson (2000).

1990s. Using data from 1980-2001, Stratmann (2006) and Stratmann and Aparicio-Castillo (2006) exploit contribution limit variations across states and confirm that stricter restrictions tend to be associated with lower campaign spending, in House and state Assembly elections, respectively.

One interpretation of the politician's preference parameter  $|\gamma|$  is the expected future campaign costs required to offset the effect of taking a policy position that is not popular with his constituents. Under this interpretation, Result 2 suggests that the effect of a more restrictive contribution cap on aggregate contributions may be quite different for House members versus Senators, as well as for members from cities versus members from rural areas. Between congressional districts there are vast differences in the cost of communicating with constituents even though they represent the same number of voters. Stratmann (2009) finds that the cost of reaching 1% of constituents with TV advertising during prime time in the 2000 election cycle ranged from \$18 in Idaho's 2<sup>nd</sup> district to \$1875 in New York City.

Since a politician from a smaller or a more rural district is likely to face a lower cost of communicating with constituents, with the same underlying policy preference the  $|\gamma|$  for this politician is likely to be lower. Result 2 implies that the politician whose preference is mild enough such that  $\gamma \in (0, v_1 - v_2]$  may see increased contributions as a result of the cap, while the politician with higher  $\gamma$  will see decreased contributions. So the cap on contributions may change the distribution of contributions between politicians. It may result in reduced contributions to Senators from larger states but increased contributions to Representatives from districts contained within minor media markets. When states consider contribution caps for state level offices, the experience with national level contribution caps may not directly apply to state politicians who generally have much lower costs of communicating with constituents.

So far the literature on the effect of caps on competition for political favors uses the amount of money in politics as the primary metric for the evaluation of campaign finance legislation. However the concern of the legislation is not that political contributions are an onerous burden on society,<sup>14</sup> but rather that they may alter the behavior of the agents involved. Hence we suggest an additional measure for the degree of influence of money on policy: The equilibrium probability that

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<sup>14</sup>Levitt (1995) points out that the amount spent on politics in the United States is roughly the same as the amount spent on chewing gum.



political contributions induce the politician to make a policy choice that he would not have made in the absence of contributions, the probability that the unfavored policy is enacted  $Prob_u$ . Note that this measure of the degree of influence of monied interests has an advantage over using expected aggregate contributions because it captures the concern that policy may be driven by money. The choice of measure matters. A more restrictive cap may lead to increased aggregate contributions while at the same time reducing the probability that the politician enacts the unfavored policy (see Figure 1).

**Result 3:** For any  $\gamma \neq 0$ ,

- (i) Any binding contribution cap reduces the influence of money on policy outcomes compared with no cap on contributions.
- (ii) The influence of money on policy outcomes is minimized with a binding but strictly positive contribution cap whenever a ban on contributions does not fully suppress all competition,  $v_u > (1 + a)|\gamma|$ .

**Proof:** Appendix B.

Figure 1 gives the probability that the unfavored policy is enacted as a function of  $m$ . While the graph is for a particular range of politician preference, as long as a complete ban on contributions does not fully suppress all competition,  $m^* < 0$ ,  $Prob_u$  has a similar U-shape, and the probability that the politician goes against his policy preference is minimized at  $m''$ .

While in equilibrium both lobbyists have a positive probability of exceeding the cap when the cap is very restrictive,  $m < m''$ , it is only the lobbyist with the unfavored policy who might violate the cap when the cap is higher than  $m''$ . When the cap restricts both competitors, a relaxation in the cap lowers both of their costs but the reduction in cost is proportionally larger for the favored lobbyist since his rival must exceed his contribution by  $|\gamma|$  in order to win. This leads to an increase in the probability that the lobbyist with the favored policy wins. When a less restrictive cap directly reduces the cost of only the unfavored lobbyist,  $m > m''$ , it leads to an increase in the unfavored lobbyist's probability of winning. Hence for all parameter values where  $m^* < 0$ , a complete ban on contributions leads to higher influence of money on policy compared to a strictly positive binding cap of  $m''$ .

The stated aim of the current legislation limiting campaign contributions is the "reduction of special interest influence." This goal may or may not be desirable, but one reasonable interpretation

of it is an attempt to reduce the probability that the politician enacts policy against his policy preference (either ideologically motivated or induced by his constituents),  $Prob_u$ . In practice however it is difficult to determine the level of the cap that minimizes the influence of money on policy outcomes. An incumbent politician is likely to have different degrees of preference across different policy issues. The valuation of the political prize to the interest groups varies depending on the policy area. Hence the legislated cap may not be able to minimize the influence of campaign contributions for all policy issues at once. However irrespective of the degree of politician preference, Result 3 shows that any binding contribution cap leads to a weaker influence of money on policy compared with no cap on contributions.

Furthermore for all policy issues where the preference is too strong,  $| \gamma | \geq \frac{a m + v}{1 + a}$  such that  $m \leq m^*$ , lobbyists do not contribute and the politician simply goes with his preference. Therefore a more restrictive cap implies a lower critical threshold of preference where there will be no influence of special interest groups on policy making. Hence politician decisions will be swayed by monied interests on a smaller number of questions.

#### **IV. Conclusion**

If the politician does not have a preference over different policy alternatives, Kaplan and Wettstein (2006) and Gale and Che (2006) find that a contribution cap is neutral on the expected total cost of lobbying and on the policy outcome as long as violations of contribution cap legislation are punished via monetary fines. We find that these neutrality results do not hold if the politician has a policy preference either based on ideology or derived from constituent preferences. A non-rigid cap with monetary punishments has an asymmetric effect on the lobbyists even if the cost functions are identical. The lobbyist with the unfavored policy is hit more heavily by the cap since he needs to contribute more than his rival in order to overcome the politician's preference. Furthermore a more restrictive cap can lead to fiercer competition and higher expected aggregate contributions and cost of contributions. Hence we retrieve the Che and Gale (1998) result without resorting to non-monetary punishments or asymmetric cost functions.

Analyzing politicians with policy preferences permits the analysis of an additional measure for the degree of influence of money on policy: The equilibrium probability that the politician makes

a policy choice that he would not have made in the absence of political contributions. The penalty for exceeding the cap may be not large enough so that a ban on contributions can fully suppress all contributions. If this is the case, a complete ban on contributions will result in a higher degree of influence of money on policy than a binding but non-zero cap.

It may be not possible to set a cap that minimizes “special interest influence” across all policy issues because the incumbent politician is likely to have different degrees of preference across issues and the valuation of the political prize will vary for the lobbyists. However Result 3(ii) shows that any binding contribution cap will reduce the influence of money on policy making compared with no cap on contributions. Hence while a cap may increase the total expected contributions, it will always decrease the influence of political contributions on policy outcomes.

However note that this paper focuses solely on the effect of contribution caps on competition for political favors. Austen-Smith (1998) analyzes the effect of caps on the incentives incumbent politicians face when deciding whether to grant access to lobbyists. While the results of both papers are encouraging for the efficacy of contribution caps, in a broader context the effect of contribution caps on electoral outcome may be perverse.<sup>15</sup> Caps may be imposed by incumbents as entry barriers to challengers in guise of promoting clean government. Challengers may be deterred from entry due to heavy filing requirements. And/or caps may make it harder to overcome the incumbent’s advantage of already being well-known.<sup>16</sup>

On the other hand, the ability to raise funds is much greater for incumbents than for challengers. In the 2006 elections the average incumbent senator raised \$11.3 million, while the average challenger raised \$1.8 million.<sup>17</sup> A contribution to buy policy favors or access is only of

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<sup>15</sup>In order to disentangle the electoral motive from buying policy favors motive of contributions and to establish clear causality between money and voting behavior, Stratmann (2002) examines repeated votes on the same piece of legislation: the repeal of provisions of the 1933 Glass-Steagall Act. The act prohibited bank holding companies from owning other financial services companies. The repeal was rejected by the House in 1991, and it then passed in 1998. It was strongly favored by banking interests but also strongly opposed by insurance and securities interests. Stratmann finds that an extra \$10,000 in contributions was associated with an 8% increase in the probability of a House member voting to repeal the prohibition.

<sup>16</sup>Lohmann (1995) points out that caps on political contributions may be counterproductive because when contributions serve as an access fee they may signal the credibility of the message.

<sup>17</sup>These of course include races with just a token challenger. Candidates for open seats, which generally include serious challengers, raised an average of \$2.8 million, substantially below the amount the average incumbent was able to raise. The figures for House races are similar, although the amounts are lower. The average incumbent raised \$1.2 million, while the average challenger raised \$283 thousand. Candidates for open seats raised an average of \$584 thousand.

potential value if the politician is in office to pay back. Over the five election cycles from 1996 to 2004 96.8% of House incumbents and 88% of the Senate incumbents were returned to office.<sup>18</sup> Hence restricting the ability to raise funds may hurt the incumbent at least as much as it hurts his challenger. However the evidence is mixed on the question of who enjoys the benefit of a cap. Hamm and Hogan (2008) finds that restrictions make the prospects of running against an incumbent more attractive to potential candidates. La Raja (2008) however reports that the financial gap widened in congressional races since BCRA. Incumbent fund-raising increased 20% between 2002 and 2006. But the challengers' did not. Stratmann *et al.*(2006) report that limits lead to closer elections but the effect is smaller on incumbents who passed the law.

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<sup>18</sup>For comparison, if these percentages stayed constant and equal for all members and there were no voluntary retirements or deaths, the expected time in office would be roughly 43 years for Senators and 50 years for Representatives.

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## APPENDIX A: Proof of Proposition 1

Equilibrium when  $\gamma=0$  is shown in KW. When  $\gamma \leq -v_2$ ,  $\gamma \geq v_1$  or  $m \leq m^*$  equilibrium is in pure strategies with neither lobbyist contributing and the prize being awarded to the lobbyist with the favored policy. Hence in what follows we restrict attention to  $m > m^*$  and  $\gamma \in (-v_2, 0) \cup (0, v_1)$ . Define for each bidder a function  $W_i(x)$  giving the bid that he must place to effectively match a bid of  $x$  from his rival. So  $W_u(x_f) = x_f + |\gamma|$  and  $W_f(x_u) = x_u - |\gamma|$ .

**Lemma 1:** Bidder  $u$  will put zero probability on  $x_i \in (0, |\gamma|]$ .

**Proof:** If bidder  $u$  contemplates  $x_u \in (0, |\gamma|)$  a bid of zero will win with the same probability as he must exceed his rival's bid by at least  $|\gamma|$  in order to win. If  $x_u = |\gamma|$  then he can win only if the other bidder bids zero, in which case there is an even chance of winning. So if that bid gave him a nonnegative payoff he could double his chances of winning by a slight increase in his bid. And if it gave him a negative payoff he could get a zero payoff by dropping his bid to zero.  $\square$

**Lemma 2:** *Neither bidder will put any probability mass point on any bid greater than zero.*

**Proof:** Suppose  $u$  had a mass point at  $x_u^* \geq |\gamma|$ . Then bidder  $f$  would not put any probability at  $x_f' = x_u^* - |\gamma|$ , either a slight increase in his bid would result in a discrete increase in the probability of winning if  $x_f' < \bar{x}_f$ , or  $x_f' \geq \bar{x}_f$  in which case bidder  $f$  would not put any probability at  $x_f'$  since by definition it would be worth bidding  $x_f'$  only if a win is certain. As there is no probability of  $x_f = x_u^* - |\gamma|$ , bidder  $u$  could lower his bid slightly without changing his probability of winning. A similar argument rules out mass points for bidder  $f$  on  $x_f > 0$ .  $\square$

**Lemma 3:** *Neither bidder will use a pure strategy.*

**Proof:** Lemma 2 rules out any pure-strategy Nash equilibrium where any bid is greater than zero. Both players' bidding zero cannot be sustained as a pure-strategy equilibrium either, since the best response to  $x_f=0$  would be for  $u$  to bid slightly higher than  $|\gamma|$  in which case it would be not optimal for  $f$  to bid zero.  $\square$

**Lemma 4:** *Bidder  $D$  has an infimum bid of zero. The expected value of the game to bidder  $D$  is zero.*<sup>19</sup>

**Proof:** Bidder  $D$ 's infimum bid must be less than  $v_D$  since there can be no probability mass at  $v_D$  by Lemma 2. Suppose that bidder  $D$  has an infimum bid of  $x_D^{\text{inf}} > 0$  such that  $W_A(x_D^{\text{inf}}) > 0$ . In that case bidder  $A$  would never bid  $W_A(x_D^{\text{inf}})$ . If he did he would be paying a positive amount and would lose for sure, since by Lemma 2, the probability of bidder  $D$  choosing exactly  $x_D^{\text{inf}}$  is zero in this range. Therefore bidder  $D$  could lower his infimum bid without changing the probability of winning. If bidder  $D$  is bidder  $f$ , then this implies directly that  $x_D^{\text{inf}} > 0$  is not possible since  $W_A(x_D^{\text{inf}}) = x_D^{\text{inf}} + |\gamma| > 0$ . However, if bidder  $D$  is bidder  $u$ , then  $W_A(x_D^{\text{inf}}) = x_D^{\text{inf}} - |\gamma|$ . So,  $W_A(x_D^{\text{inf}}) > 0$  for  $x_D^{\text{inf}} > |\gamma|$ .

Suppose that bidder  $D$  is bidder  $u$  and  $b_u^{\text{inf}} = |\gamma|$  where bidder  $u$  was mixing in the open interval above  $|\gamma|$  but not at  $|\gamma|$ , by Lemma 1. Then bidder  $f$  would never bid zero as this gives zero profit and he can win for sure with a bid of  $W_f(\bar{x}_u)$  yielding a positive profit. Take a bid of

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<sup>19</sup>Siegel (2009) could be used to directly establish the expected values. However since this game is not generic for all sets of parameter values Theorem 1 does not apply and only the weaker Corollary 2 can be used. Hence it would not be possible to establish the uniqueness of the equilibrium distribution functions using that approach.

$b_u = b_u^{\text{inf}} + \varepsilon = |\gamma| + \varepsilon$ , the probability that bidder  $u$  wins with this bid is  $\int_{|\gamma|}^{|\gamma|+\varepsilon} f_f(x-|\gamma|)dx$ . Since bidder  $f$  has no mass points on  $(0, \varepsilon]$  by Lemma 2, this probability is close to zero for small  $\varepsilon$ , yielding a negative expected payoff for bidder  $u$ . Hence bidder  $u$ 's infimum bid cannot be  $|\gamma|$ .  $b_u^{\text{inf}} \in (0, |\gamma|)$  is not possible by Lemma 1. Therefore a bid of zero is in the support of the mixed strategy of bidder  $u$ .

Therefore a bid of zero is in the support of the mixed strategy of bidder  $D$ . At this bid he loses for sure, so the expected value of the strategy for bidder  $D$  must be zero.  $\square$

**Lemma 5:** *Bidder  $D$  has a supremum bid of  $x_D^{\text{sup}} = \bar{x}_D$ . Bidder  $A$  has a supremum bid of  $x_A^{\text{sup}} = W_A(\bar{x}_D)$ . The expected value of the game to bidder  $A$  is  $v_A - c(W_A(\bar{x}_D)) \geq 0$ , with equality only if bidder  $u$  is the advantaged bidder  $A$  and  $\bar{x}_A = \bar{x}_D + |\gamma|$ .*

**Proof:** Bidder  $D$  cannot have supremum bid of zero since that would be a pure strategy which contradicts Lemma 3. Suppose that bidder  $D$  has a supremum bid of  $x_D^{\text{sup}} \in (0, \bar{x}_D)$ . Then bidder  $A$  would never set  $x_A > W_A(x_D^{\text{sup}})$  as he can win for sure with  $x_A = W_A(x_D^{\text{sup}})$  since by Lemma 2 the probability of bidder  $D$  choosing exactly  $x_D^{\text{sup}}$  is zero. Therefore bidder  $D$  could win for sure with  $x_D = x_D^{\text{sup}} + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 4.

Suppose that bidder  $A$  has a supremum bid of  $x_A^{\text{sup}} < W_A(\bar{x}_D)$ . Then bidder  $D$  could lower his supremum bid and still win for sure with that bid yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 4. Bidder  $A$  can win for sure with a bid of  $W_A(\bar{x}_D)$  since by Lemma 2 the probability of bidder  $u$  choosing exactly  $\bar{x}_D$  is zero. Since  $W_A(\bar{x}_D)$  is in the support of his mixed strategy and he wins for sure with that bid, the expected payoff for bidder  $A$  is  $v_A - c(W_A(\bar{x}_D))$ .

If bidder  $f$  is the advantaged bidder  $A$ , then  $W_A(\bar{x}_D) = \bar{x}_D - |\gamma|$  and the definition of the advantaged bidder yields  $\bar{x}_A > \bar{x}_D - |\gamma|$  and so the expected payoff of bidder  $A$  is strictly positive. If bidder  $u$  is the advantaged bidder  $A$ , then  $W_A(\bar{x}_D) = \bar{x}_D + |\gamma|$  and the definition of the advantaged bidder yields  $\bar{x}_A \geq \bar{x}_D + |\gamma|$  so the expected payoff of bidder  $A$  is strictly positive unless  $\bar{x}_A = \bar{x}_D + |\gamma|$  in which case it is zero.  $\square$

**Lemma 6:** *For bidder  $f$ , bids almost everywhere on  $x_f \in [0, x_f^{\text{sup}}]$  and for bidder  $u$ , bids almost everywhere on  $x_u \in (|\gamma|, x_u^{\text{sup}}]$ , must have positive probability.*

**Proof:** Suppose there were an interval  $(t, s)$  in  $(|\gamma|, x_u^{\text{sup}}]$  where bidder  $u$  had zero probability of bidding. Then bidder  $f$  would have zero probability of bidding in  $(t-|\gamma|, s-|\gamma|)$  since he could lower his bid to  $t-|\gamma|$  and have the same chance of winning. But in this case bidder  $u$  would never bid  $s + \varepsilon$  as he could lower his bid to  $t$ , saving  $s + \varepsilon - t$  in bidding costs and losing only  $F_f(s + \varepsilon - |\gamma|) - F_f(t - |\gamma|)$  in probability. By Lemma 2 the loss in probability is negligible for small  $\varepsilon$ . So if there is an interval of zero probability it must go up to  $x_u^{\text{sup}}$  which contradicts Lemma 5. A symmetric argument rules out ranges of zero probability for bidder  $f$  in  $x_f \in [0, x_f^{\text{sup}}]$ .  $\square$

**Lemma 7:** If  $\gamma < 0$  or  $\gamma > v_1 - v_2$  then  $\bar{x}_u - \bar{x}_f < |\gamma|$ . If  $\gamma \in \left(0, \frac{v_1 - v_2}{1+a}\right]$  then  $\bar{x}_u - \bar{x}_f \geq |\gamma|$ . If  $\gamma \in \left(\frac{v_1 - v_2}{1+a}, v_1 - v_2\right]$  then  $\bar{x}_u - \bar{x}_f \geq |\gamma|$  when  $m \geq \tilde{m} = \frac{(1+a)(v_2 + \gamma) - v_1}{a}$  and  $\bar{x}_u - \bar{x}_f < |\gamma|$  if  $m$  is lower.

**Proof:** Consider first  $\gamma < 0$ . Then  $f$  is bidder 1 and  $u$  is bidder 2. If  $m \geq v_1$  then  $\bar{x}_1 = v_1$  and  $\bar{x}_2 = v_2$ . If  $v_2 < m < v_1$  then  $\bar{x}_1 = \frac{v_1 + am}{1+a}$  and  $\bar{x}_2 = v_2$ . If  $m \leq v_2$  then  $\bar{x}_1 = \frac{v_1 + am}{1+a}$  and  $\bar{x}_2 = \frac{v_2 + am}{1+a}$ . For all ranges of  $m$ ,  $\bar{x}_1 > \bar{x}_2$ . Hence  $\bar{x}_u - \bar{x}_f < 0 < |\gamma|$ .

Now consider  $\gamma > v_1 - v_2$ , a positive  $\gamma$  implies  $f=2$  and  $u=1$ . Hence if  $m \geq v_2$ ,  $\bar{x}_f = v_2$  and  $\bar{x}_u \leq v_1$ . So  $\bar{x}_u - \bar{x}_f \leq v_1 - v_2 < |\gamma|$ . If  $m < v_2$  then  $\bar{x}_u = \frac{v_1 + am}{1+a}$  and  $\bar{x}_f = \frac{v_2 + am}{1+a}$  so  $\bar{x}_u - \bar{x}_f = \frac{v_1 - v_2}{1+a} < v_1 - v_2 \leq |\gamma|$ .

Finally consider  $\gamma \in \left(0, v_1 - v_2\right]$ . Again  $f=2$  and  $u=1$ . Hence if  $m \geq v_1$ ,  $\bar{x}_f = v_2$  and  $\bar{x}_u = v_1$ . So  $\bar{x}_u - \bar{x}_f = v_1 - v_2 \geq |\gamma|$ . If  $m < v_2$  then  $\bar{x}_u = \frac{v_1 + am}{1+a}$  and  $\bar{x}_f = \frac{v_2 + am}{1+a}$  so  $\bar{x}_u - \bar{x}_f = \frac{v_1 - v_2}{1+a}$  which is less than  $\gamma$  if  $\gamma > \frac{v_1 - v_2}{1+a}$  and greater than or equal to  $\gamma$  otherwise. If  $m \in [v_2, v_1 - v_2]$  then  $\bar{x}_f = v_2$  and  $\bar{x}_u = \frac{v_1 + am}{1+a}$  so  $\bar{x}_u - \bar{x}_f = \frac{v_1 + am}{1+a} - v_2$ . This is  $< \frac{(1+a)(v_2 + \gamma) - v_1}{a}$  and greater than or equal to  $\gamma$  otherwise. If  $\gamma \leq \frac{v_1 - v_2}{1+a}$  then this expression does not hold for any  $m \geq v_2$  and so  $\bar{x}_u - \bar{x}_f \geq |\gamma|$ .  $\square$

**Lemma 8:** The unique equilibrium distribution functions are those given in Proposition 1.

**Proof:** Part (i): By lemma 7,  $\bar{x}_f > \bar{x}_u - |\gamma|$ . In this case the favored bidder  $f$  is also the advantaged bidder  $A$ . By lemmas 5 and 6 all bids  $x_f \in [0, W_f(\bar{x}_u)]$  must yield an expected payoff of  $v_f - c(W_f(\bar{x}_u))$  to bidder  $f$ . By lemma 2 in this range there is zero probability that  $x_u = x_f + |\gamma|$ . So bidder  $f$  wins if  $x_f \geq x_u - |\gamma|$ . Therefore on that range:

$v_f - c(W_f(\bar{x}_u)) = v_f F_u(x + |\gamma|) - c(x)$   
So from equation 1 and the fact that  $W_f(\bar{x}_u) = \bar{x}_u - |\gamma|$ :

$$v_f - c(\bar{x}_u - |\gamma|) = \begin{cases} v_f F_u(x + |\gamma|) - x & \text{for } x \in (0, \min(m, \bar{x}_u - |\gamma|)] \\ v_f F_u(x + |\gamma|) - (1+a)x + am & \text{for } x \in (m, \bar{x}_u - |\gamma|] \end{cases}$$

letting  $x_u = x + |\gamma|$  and solving for  $F_u(\cdot)$ :



$$F_u(x_u) = \begin{cases} \frac{v_f - \bar{x}_u + x_u}{v_f} & \text{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_u)] \\ \frac{v_f - \bar{x}_u - a(m + |\gamma|) + (1+a)x_u}{v_f} & \text{for } x_u \in (m + |\gamma|, \bar{x}_u] \end{cases}$$

Bidder  $u$  places no probability on  $(0, |\gamma|]$  by lemma 1 so plugging in  $|\gamma|$  at the bottom of the range yields the probability mass at zero.

By lemmas 4, 5 and 6 all bids in  $(|\gamma|, \bar{x}_u]$  yield an expected payoff of zero to bidder  $u$ . By lemma 2 in this range there is zero probability that  $x_f = x_u - |\gamma|$  so bidder  $u$  wins if  $x_u \geq x_f + |\gamma|$ . Therefore on that range:

$$0 = v_u F_f(x - |\gamma|) - c(x)$$

So from equation 1

$$0 = \begin{cases} v_u F_f(x - |\gamma|) - x & \text{for } x \in (|\gamma|, \min(m, \bar{x}_u)) \\ v_u F_f(x - |\gamma|) - (1+a)x + am & \text{for } x \in [m, \bar{x}_u] \end{cases}$$

letting  $x_f = x - |\gamma|$  and solving for  $F_f(\cdot)$ :

$$F_f(x_f) = \begin{cases} \frac{|\gamma| + x_f}{v_u} & \text{for } x_f \in (0, \min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|)) \\ \frac{(1+a)|\gamma| - am + (1+a)x_f}{v_u} & \text{for } x_f \in [\max(0, m - |\gamma|), \bar{x}_u - |\gamma|) \end{cases}$$

plugging in zero at the bottom of the range yields the probability mass at zero.

*Part (ii):* By lemma 7,  $\bar{x}_u \geq \bar{x}_f + |\gamma|$ . In this case the unfavored bidder  $u$  is the advantaged bidder  $A$ . By lemmas 4 and 6 all bids in  $[0, \bar{x}_f]$  must yield an expected payoff of zero to bidder  $f$ . By lemma 2 in this range there is zero probability that  $x_u = x_f + |\gamma|$ . So bidder  $f$  wins if  $x_f \geq x_u - |\gamma|$ . Therefore on that range:

$$0 = v_f F_u(x + |\gamma|) - c(x)$$

So from equation 1:

$$0 = \begin{cases} v_f F_u(x + |\gamma|) - x & \text{for } x \in (0, \min(m, \bar{x}_f)] \\ v_f F_u(x + |\gamma|) - (1+a)x + am & \text{for } x \in (m, \bar{x}_f] \end{cases}$$

letting  $x_u = x + |\gamma|$  and solving for  $F_u(\cdot)$ :

$$F_u(x_u) = \begin{cases} \frac{x_u - |\gamma|}{v_f} & \text{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_f + |\gamma|)] \\ \frac{-(1+a)|\gamma| - am + (1+a)x_u}{v_f} & \text{for } x_u \in (m + |\gamma|, \bar{x}_f + |\gamma|] \end{cases}$$

Bidder  $u$  places no probability on  $(0, |\gamma|]$  by lemma 1 so plugging in  $|\gamma|$  at the bottom of the range yields the probability mass at zero.

By lemmas 4, 5 and 6 all bids in  $(|\gamma|, W_u(\bar{x}_f)]$  yield an expected payoff of  $v_u - c(W_u(\bar{x}_f))$  to bidder  $u$ . By lemma 2 in this range there is zero probability that  $x_f = x_u - |\gamma|$  so bidder  $u$  wins if  $x_u \geq x_f + |\gamma|$ . Therefore on that range:

$$v_u - c(W_u(\bar{x}_f)) = v_u F_f(x - |\gamma|) - c(x)$$

So equation 1 and the fact that  $W_u(\bar{x}_f) = \bar{x}_f + |\gamma|$  yield:

$$v_u - c(\bar{x}_f + |\gamma|) = \begin{cases} v_u F_f(x - |\gamma|) - x & \text{for } x \in (|\gamma|, \min(m, \bar{x}_f + |\gamma|)) \\ v_u F_f(x - |\gamma|) - (1+a)x + am & \text{for } x \in [m, \bar{x}_f + |\gamma|) \end{cases}$$

letting  $x_f = x - |\gamma|$  and solving for  $F_f(\cdot)$ :

$$F_f(x_f) = \begin{cases} \frac{v_u - c(\bar{x}_f + |\gamma|) + |\gamma| + x_f}{v_u} & \text{for } x_f \in (0, \min(m - |\gamma|, \bar{x}_f)) \\ \frac{v_u - c(\bar{x}_f + |\gamma|) - am + (1+a)(x_f + |\gamma|)}{v_u} & \text{for } x_f \in (m - |\gamma|, \bar{x}_f] \end{cases}$$

plugging in zero at the bottom of the range yields the probability mass at zero.  $\square$

**Lemma 9:** *The equilibrium distributions, and hence all comparative statics, are continuous in  $m$ .*

**Proof:** The distributions in each part of Proposition 1 are clearly continuous in  $m$ . When  $\gamma \in \left(\frac{v_1 - v_2}{1+a}, v_1 - v_2\right]$  the distribution switches from that of Proposition 1 (i) to that of Proposition 1

(ii) at  $m = \frac{(1+a)(v_2 + \gamma) - v_1}{a}$ . However at that point the distributions are identical. If

$a \leq [v_u - |\gamma|] / |\gamma|$ , then all competition is curtailed at  $m^* = [(1+a)|\gamma| - v_u] / a$ . At that point the favored bidder is advantaged so the distribution of Proposition 1 (i) applies and it results in no competition between the bidders. The implication of Lemma 9 is that making a cap more restrictive will never result in a discontinuous jump in the expected bids as it does in CG.  $\square$

This completes the proof of Proposition 1.

## APPENDIX B: Proof of Results

The results follow directly from Proposition 1. First we show how to use the relative levels of the critical values of  $m$  to determine the min and max arguments in the equilibrium distribution functions in Proposition 1. It is then straightforward to calculate  $Prob_u$ ,  $E(Bids)$  and  $E(Costs)$  from the equilibrium distribution functions in Proposition 1:

$$Prob_u = \int_u F_f(x - |\gamma|) f_u(x) dx$$

$$E(Bids) = \int_u x f_u(x) dx + \int_f x f_f(x) dx$$

$$E(\text{Costs}) = E(\text{Bids}) + \int_{\min(m, x_u^{\text{sup}})}^{x_u^{\text{sup}}} a(x-m)f_u(x)dx + \int_{\min(m, x_f^{\text{sup}})}^{x_f^{\text{sup}}} a(x-m)f_f(x)dx$$

Table 1 presents comparative statics for these measures for each of the possibilities for  $m$  relative to the critical values of  $m$  for  $m \in (\max(0, m^*), \bar{m})$ . This information is then used to prove the results.

**Determining the min max arguments in Proposition 1:**

This is considered in two parts corresponding to the two parts of Proposition 1. For all distribution functions, note that  $\max(0, m-|\gamma|)=0$  if  $m < |\gamma|$ .

*i)* When Proposition 1(i) applies and  $m^* < m \leq \bar{m}$  the ranges in the favored lobbyist's distribution function depend on whether or not  $m < |\gamma|$ :

$$\text{if } m \leq |\gamma|, \min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = 0$$

$$\text{if } m > |\gamma|, \min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = m - |\gamma|$$

while the ranges in the unfavored lobbyist's distribution function depend on whether or not  $m < m''$ :

$$\text{if } m \leq m'', \min(m+|\gamma|, \bar{x}_u) = m+|\gamma|$$

$$\text{if } m > m'', \min(m+|\gamma|, \bar{x}_u) = \bar{x}_u$$

When Proposition 1 applies,  $m'' = v_u - (1+a)|\gamma|$  and  $\bar{x}_u = (v_u + am)/(1+a)$ .

*(ii)* When Proposition 1(ii) applies, and  $m^* < m \leq \bar{m}$  the ranges in the favored lobbyist's distribution function depend on whether or not  $m < |\gamma|$ :

$$\text{if } m \leq |\gamma|, \min(\max(0, m-|\gamma|), \bar{x}_f) = 0$$

$$\text{if } m > |\gamma|, \min(\max(0, m-|\gamma|), \bar{x}_f) = m - |\gamma|$$

while the ranges in the unfavored lobbyist's distribution function depend on whether or not  $m < m''$ :

$$\text{if } m \leq m'', \min(m+|\gamma|, \bar{x}_f + |\gamma|) = m+|\gamma|$$

$$\text{if } m > m'', \min(m+|\gamma|, \bar{x}_f + |\gamma|) = \bar{x}_f + |\gamma|$$

When Proposition 1 (ii) applies,  $m'' = v_f$  and  $\bar{x}_f = \begin{cases} v_f & \text{if } m > m'' \\ \frac{v_f + am}{1+a} & \text{if } m \leq m'' \end{cases}$

Table 1: Comparative Statics when  $m \in (m^*, \bar{m})$

Proposition 1(i) applies and		$\frac{dE(Bids)}{dm}$	$\frac{dE(Costs)}{dm}$	$\frac{d Prob_u}{dm}$
I	$m <  \gamma $ and $m < m''$	$\frac{a(\bar{x}_u -  \gamma  - m)}{v_f} + \frac{a(\bar{x}_u -  \gamma )}{v_u} > 0$	$am \left( \frac{v_f - v_u}{v_u v_f} \right) = \begin{cases} > 0 \text{ if } f = 1 \\ < 0 \text{ if } f = 2 \end{cases}$	$\frac{-am}{v_u v_f} < 0$
II	$m <  \gamma $ and $m > m''$	$\frac{a\bar{x}_u}{(1+a)v_f} + \frac{a(\bar{x}_u -  \gamma )}{v_u} > 0$	$\frac{a(\bar{x}_u -  \gamma )}{v_u} + \frac{a \gamma  + a^2( \gamma  - m)}{(1+a)v_f} > 0$	$\frac{a( \gamma  + a( \gamma  - m))}{(1+a)v_u v_f} > 0$
III	$m >  \gamma $ and $m < m''$	$\frac{a(\bar{x}_u -  \gamma  - m)}{v_f} + \frac{a(\bar{x}_u - m)}{v_u} > 0$	$a \gamma  \left( \frac{v_f - v_u}{v_u v_f} \right) = \begin{cases} > 0 \text{ if } f = 1 \\ < 0 \text{ if } f = 2 \end{cases}$	$\frac{-a \gamma }{v_u v_f} < 0$
IV	$m >  \gamma $ and $m > m''$	$\frac{a\bar{x}_u}{(1+a)v_f} + \frac{a(\bar{x}_u - m)}{v_u} > 0$	$\frac{a(\bar{x}_u - m)}{v_u} + \frac{am}{(1+a)v_f} > 0$	$\frac{am}{(1+a)v_u v_f} > 0$
Proposition 1(ii) applies and:				
V	$m <  \gamma $ and $m < m''$	$\frac{a(\bar{x}_f - m)}{v_f} + \frac{a\bar{x}_f}{v_u} > 0$	$am \left( \frac{v_f - v_u}{v_u v_f} \right) = \begin{cases} > 0 \text{ if } f = 1 \\ < 0 \text{ if } f = 2 \end{cases}$	$\frac{-am}{v_u v_f} < 0$
VI	$m <  \gamma $ and $m > m''$	0	$-a < 0$	0
VII	$m >  \gamma $ and $m < m''$	$\frac{a(\bar{x}_f - m)}{v_f} + \frac{a(\bar{x}_f - m +  \gamma )}{v_u} > 0$	$a \gamma  \left( \frac{v_f - v_u}{v_u v_f} \right) = \begin{cases} > 0 \text{ if } f = 1 \\ < 0 \text{ if } f = 2 \end{cases}$	$\frac{-a \gamma }{v_u v_f} < 0$
VIII	$m >  \gamma $ and $m > m''$	$\frac{-a(m -  \gamma )}{v_u} < 0$	$\frac{a(m - (v_f +  \gamma ))}{v_f} - \frac{a(m -  \gamma )}{v_u} < 0$	$\frac{a(m -  \gamma )}{v_u v_f} > 0$

**Proof of Result 1:** From Table 1  $\forall \gamma \in (-v_2, 0) \cup (0, v_1)$  and  $m \in (\max(0, m^*), \bar{m})$  changes in  $m$  are never neutral on expected costs.  $\square$

**Proof of Result 2:** When  $\gamma \in \left(0, \frac{v_1 - v_2}{1+a}\right]$  Proposition 1(ii) applies. It also applies when  $\gamma \in \left[\frac{v_1 - v_2}{1+a}, v_1 - v_2\right]$  and  $m > \tilde{m}$ . In this later case from (6)  $m'' = v_2$  and  $\tilde{m} > m''$  since  $\gamma > (v_1 - v_2)/(1+a)$ . So in either case  $m > \max(m'', \gamma)$  so Table 1 row VIII yields the result.  $\square$

**Proof of Result 3:** When  $v_u > (1+a)|\gamma|$  from (3)  $m^* < 0$  and from (4) and (6)  $m'' \in (0, \bar{m})$ . From Proposition 1,  $Prob_u$  is continuous in  $m$  and Table 1 shows that  $Prob_u$  is U-shaped: Decreasing in  $m$  for all  $m \in (0, m'')$  and increasing in  $m$  for all  $m \in (m'', \bar{m})$ . This completes the proof of part (ii). It also shows that whenever  $m^* < 0$  then  $Prob_u$  will be maximized at one of the endpoints, either  $m=0$  or  $m = \bar{m}$ .

If  $\gamma \in \left(0, (v_1 - v_2)/(1+a)\right]$  then  $m^* \geq 0$  is not possible so from (4) and (6) whenever  $m^* \geq 0$ ,  $m'' = v_u - (1+a)|\gamma| \leq 0$ . Hence  $m > m'' \forall m \in (m^*, \bar{m})$  and thus Table 1 shows that  $Prob_u$  is weakly increasing in  $m$  over this entire range. Moreover (4) implies that  $\bar{m} > |\gamma|$  so range VI in Table 1 (where there is no effect of changes in  $m$  on  $Prob_u$ ) must be strictly below  $\bar{m}$ . Thus if  $m^* \geq 0$  any level of cap  $m < \bar{m}$  will reduce  $Prob_u$  compared to no contribution cap.

It only remains to show that when  $m^* < 0$  the level of  $Prob_u$  is weakly lower at  $m=0$  than at  $m = \bar{m}$ . Without a cap, when  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$  then Proposition 1 part (i) applies and,

$$Prob_u = \int_u F_f(x - |\gamma|) f_u(x) dx = \frac{1}{2v_u v_f} (v_u^2 - \gamma^2) \quad (8)$$

When  $m=0$  and  $\gamma \in (-v_2, 0) \cup \left(\frac{v_1 - v_2}{1+a}, v_1\right)$  then Proposition 1 part (i) applies and

$$Prob_u = \int_u F_f(x - |\gamma|) f_u(x) dx = \frac{1}{2v_u v_f} [v_u^2 - (1+a)^2 \gamma^2] \quad (9)$$

which is strictly less than (8) and so part (i) holds when  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ . When  $\gamma \in (0, v_1 - v_2]$  and there is no cap then Proposition 1 part (ii) applies and

$$Prob_u = \int_u F_f(x - |\gamma|) f_u(x) dx = 1 - \frac{v_f}{2v_u} \quad (10)$$

When  $\gamma \in \left(\frac{v_1 - v_2}{1+a}, v_1 - v_2\right]$   $Prob_u$  is given by (9) when  $m=0$  which is strictly less than (10) for  $\gamma > (v_1 - v_2)/(1+a)$  so part (i) holds for this range of  $\gamma$ . Finally, when  $m=0$  and  $\gamma \in \left(0, (v_1 - v_2)/(1+a)\right]$  then Proposition 1 part (ii) applies so

$$Prob_u = \int_u F_f(x - |\gamma|) f_u(x) dx = 1 - \frac{v_f}{2v_u} \quad (11)$$

which is equal to (10) so for this final range of  $\gamma$  part (ii) holds weakly for a complete ban on contributions and holds strictly for any  $\gamma \in (0, \bar{m})$ .  $\square$