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Exploring Long Memory and Nonlinearity in Irish Real Exchange Rates using Tests based on Semiparametric Estimation

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Abstract

Deciding whether a time series that appears nonstationary is in fact fractionally integrated or subject to structural change is a difficult task. However, various tests have recently been introduced for distinguishing long memory from level shifts and nonlinearity. In this paper, three testing approaches based on the properties of semiparametric estimators of the fractional differencing parameter, d, are described and applied to the (log) Ireland-United Kingdom and Ireland-Germany real exchange rates. The two exchange rates behave quite differently over time and the new tests give different results for each; but overall the results provide fairly strong support for the possibility of nonlinearity rather than long memory.

J.E.L. Classification: C22, C51, F31

Keywords: Fractional integration, long memory, nonlinearity, real exchange rates, structural change.

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1 Introduction

In a recent paper, Bond, Harrison, and O'Brien (2007) investigated the issue of structural breaks and nonlinearities in the modelling of Irish purchasing power parity (PPP) relationships. Their preliminary analysis, based on the fractional augmented Dickey-Fuller (FADF) test of Dolado, Gonzalo, and Mayoral (2002), suggested the possibility that both the nominal and the real exchange rate data used were generated by long memory processes. By contrast, using the three-stage random field approach introduced by Hamilton (2001), they found strong evidence of nonlinearity when PPP was modelled using nominal exchange rates but little evidence when using real exchange rates. This paper investigates these apparently conflicting results further, using three recently introduced approaches to testing for nonlinearity versus long memory.

Nonlinear behaviour and persistence in real exchange rates are major areas of PPP research (see, for example, Sarno (2005), and Rogoff (1996) and Villeneuve and Handa (2006)) and the emphasis on real exchange rates has allowed the use of univariate techniques such as smooth transition autoregressive (STAR) models and their associated test procedures. The three approaches used here result from the surge of interest in the possibility of confusing long memory processes with stationary short memory processes subject to structural change (see, for instance, Shimotsu (2006) and Perron and Qu (2008)) and augment the univariate approach. However, unlike standard tests for nonlinearity, such as RESET and STAR-F tests, the new tests employ semiparametric rather than parametric estimators. Specifically, they are based on the behaviour of semiparametric estimates of the fractional integration parameter, d, when data are actually generated by stationary processes contaminated by structural shifts. The first approach uses the modified Geweke and Porter-Hudak (1983) (*modified-GPH*) estimator of d due to Smith (2005); the second, proposed by Perron and Qu (2008), uses the *GPH* estimator of d; and the third, suggested by Shimotsu (2006), uses the *Whittle* estimator. In the present context, each of these procedures may be viewed as a test for spurious long memory and, therefore, as a useful means of checking the Bond, et al. (2007) findings.

The structure of the paper is as follows. Section 2 summarises the tentative results on PPP for Ireland vis-à-vis the United Kingdom (UK) and Germany reported by Bond, et al. (2007) and describes the data on which these results are based. Section 3 outlines the three approaches used in this study, and Section 4 presents and discusses the results of applying these methods to the real exchange rates used in Bond, et al. (2007). Section 5 concludes the paper.

2 Background

Put simply, the PPP hypothesis is that when expressed in a common currency, national prices should be equal. It is generally accepted that this hypothesis holds in the long run. However, using traditional regression analysis, little evidence has been found to support it. In recent years, studies focusing on the nonstationarity of the various time series involved have dominated the literature, although works such as Maynard (2006), which uses tests robust to persistence in conditioning variables, suggest that even if nonstationarity is considered, a substantial economic puzzle remains.

It is well known (see Perron 1989) that it is difficult to distinguish statistically between nonstationary (unit root, $d = 1$) linear processes and stationary but nonlinear processes. This phenomenon is now known to apply in the long memory ($0 < d < 1$) case as well; see, for example, Gourieroux and Jasiak (2001), and Granger and Hyung (2004). The idea that real exchange rates are nonlinear has a long history (see Taylor 2001, for discussion). Nonlinearity may arise in exchange rate data for several economic reasons, including transactions costs, central bank interventions and the existence of limits to speculation; see Taylor (2006). The challenge is deciding how to model the nonlinearities. Early attempts used Markov-switching models (Engel and Hamilton 1990). More recently, smooth transition autoregression has become popular; see, for example, Sarno, Valente, and Hyginus (2004) and Baillie and Kiliç (2005). The problem with these approaches is that they assume the form of the nonlinearity is known. In an attempt to overcome this, Bond, et al. (2007) used the three-step random field regression analysis of Hamilton (2001) to explore the nature of nonlinearity. A shortcoming of the random field approach in univariate modelling is that assumptions have to be made about the autoregressive nature of the series.

Bond, et al. (2007), using 115 quarterly data observations for Ireland, Germany and the UK for the period 1975 Q1 to 2003 Q3, explored both a causal PPP model of the form

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \epsilon_t, \tag{1}$$

and the univariate log real exchange rate series

$$q_t = s_t + p_t - p_t^*, \tag{2}$$

where s_t is the logarithm of the nominal exchange rate, p_t and p_t^* are the logarithms of the domestic and foreign price levels, respectively, and ϵ_t is a white noise disturbance. They used both parametric structural break tests, such as those of Bai and Perron (2003), and random field inference, following Hamilton (2001) and Dahl and González-Rivera (2003). They found strong support for nonlinearity in the casual model but little support for nonlinearity of the real exchange rates.[1] However, using the FADF test, they found some support for the hypothesis that real exchange rates are fractionally integrated series.

It is these tantalising findings, and the increasing interest in using the properties of estimates of the fractional integration parameter, d, to investigate the possibility of nonlinearity in series that appear to be generated by long memory processes, that motive the present paper. The three approaches of Smith (2005), Shimotsu (2006) and Perron and Qu (2008) may shed some light on the possibility that the apparent long memory behaviour of the Ireland-Germany and Ireland-UK real exchange rates could be due in fact to nonlinearity.

Figure 1 shows the simple time-series plots of the Irish real exchange rate relative to the UK and the rate relative to Germany. The start of the data period pre-dates the European Monetary System and the break of the Irish Punt with Sterling, so the Sterling/Irish Punt nominal exchange rate was constant from 1975 until 1978. Likewise, as a result of European Monetary Union and the introduction of the Euro, the nominal Deutsche-Mark/Irish Punt rate has been constant since 1999. The plots show that the two real exchange rates have quite different historical patterns. The real exchange rate between Ireland and Germany is one of an initially declining rate that has, especially in recent years, levelled off, while that between Ireland and the UK is more complex, with periods of little change followed by periods of major movements. Given the very different behaviour of the two time series, it is likely that the tests to be used will perform differently.

3 The Tests

The test of Smith (2005) is based on his *modified-GPH* estimator, which exploits how the bias of the *GPH* estimator of d performs when the underlying model is nonlinear rather than fractional. The Shimotsu (2006) approach actually provides a suite of tests that makes use of how *Whittle* estimates of d vary in subsamples when the underlying model is nonlinear, and also how the d-differenced series should be $I(0)$ if the model is linear and fractional. The Perron and Qu (2008) test is based on how the *GPH* estimator of d varies as the number of frequencies used changes and the model is a short memory process subject to level shifts. There follows a brief outline of each procedure providing a few further details.

[1]Here, and in the rest of the paper, 'real exchange rate' is used to refer to the log real exchange rate, q_t, defined in (2).

3.1 The *modified-GPH* test

Smith (2005) considers the properties of d, estimated (incorrectly) from a fairly general Mean-plus-Noise (MN) model, which has the general form

$$y_t = \mu_t + \epsilon_t \qquad t = 1, 2, ..., T, \tag{3}$$

and

$$\mu_t = (1 - p)\mu_{t-1} + \sqrt{p}\eta_t \qquad 0 < p < 1, \tag{4}$$

where ϵ_t and η_t are short-memory random variables each with zero mean and finite nonzero variance, and ϵ_t and η_s are independent of each other for all t and s. The parameter p determines the persistence of the level component μ_t. This MN specification encompasses models such as Markov-switching and stationary random level shift models.

The *GPH* estimator of d for the MN model (3) and (4), say $\hat{d}$, is consistent under standard Gaussian assumptions but, as Smith (2005) shows, it is biased upwards. By exploring the nature of this bias, Smith (2005) derives a modified version of the *GPH* estimator that has a smaller bias. The modification is essentially the addition of another term to the *GPH* regression. If $\hat{f}_j$, $j = 1, 2, \ldots, m$, is the periodogram, the modified regression is

$$\log \hat{f}_j = \alpha + dX_j + \beta Z_{kj} + \hat{u}_j, \tag{5}$$

where X_j is the standard *GPH* term

$$X_j = -\log(2 - 2cos(\omega_j)) \qquad \omega_j = \pi j/T,$$

and Z_{kj} is the additional term

$$Z_{kj} = -\log\left((kj)^2 / T^2 + \omega_j^2\right),$$

and k is a nuisance parameter, which Smith (2005) suggests has a value between one and five. Smith (2005) also shows that in many circumstances a value of $k = 3$ is optimal.

The *modified-GPH* estimator, say $\hat{d}^k$, can be used to investigate whether the apparent fractional nature of a series is really due to mean shift. If $\hat{d}^k < \hat{d}$, then it is likely that the series contains a mean shift. If $\hat{d}^k \geqslant \hat{d}$, then it is unlikely that the evidence for fractional behaviour is due to mean shifts. Importantly, Smith (2005) points out that $\hat{d}^k$ should not be viewed as an estimate of the 'true' value of d as this requires nontrivial modelling. It should also be noted that there are no critical values for Smith's procedure; it is not a formal significance test but rather a useful diagnostic check.

A major issue when calculating *GPH* estimates is the choice of the number of frequencies, m. Increasing m normally leads to a smaller root mean square error but larger bias (Hurvich, Deo, and Brodsky 1999). Smith (2005) uses the rule-of-thumb fixed value of $m = T^{1/2}$, suggested by Geweke and Porter-Hudak (1983) and the 'Plugin', root mean square minimising value suggested by Hurvich and Deo (1999).

3.2 The $t_d(a, c_1, b, c_2)$ test

Perron and Qu (2008) explore the behaviour of $\hat{d}$ as m varies. Using their theoretical results and simulation findings for a short memory level shift model, they propose three related tests of the null hypothesis, H_0, of long memory that have power against such an alternative stationary process. The tests are based on the difference in $\hat{d}$ using different values of m. If the time series in question is a true long memory series, the values of $\hat{d}$ should not vary greatly as m changes. However, if the series is a short memory process subject to level shifts, the values of $\hat{d}$ follow a particular pattern. When m is near $T^{1/3}$, $\hat{d}$ will be close to one. As m increases from $T^{1/3}$ to $T^{1/2}$, the stationary component begins to have more effect and so the values of $\hat{d}$ decline. The exact nature of the decline depends on the underlying process. After $m = T^{1/2}$, the estimate $\hat{d}$ continues to decline gradually.

The basic test statistic is:

$$t_d(a, c_1; b, c_2) = \sqrt{\frac{24c_1\left[T^a\right]}{\pi^2}}\left(\hat{d}_{a,c_1} - \hat{d}_{b,c_2}\right), \tag{6}$$

where $0 < a < b < 1$, and $\hat{d}_{a,c_1}$ and $\hat{d}_{b,c_2}$ are the *GPH* estimates corresponding to $m_a = c_1\left[T^a\right]$ and $m_b = c_2\left[T^b\right]$, respectively, where $[x]$ denotes the integer part of x. Under the null hypothesis that the series is $I(d)$ with $0 < d < \frac{1}{2}$, and with $b = \frac{4}{5}$, $t_d(a, c_1; b, c_2)$ is asymptotically distributed as $N(0, 1)$.

The first test suggested by Perron and Qu (2008) is $t_d\left(\frac{1}{2}, 1, \frac{4}{5}, 1\right)$. This is a test of whether the estimate of d declines gradually over the range $[T^{1/2}, T^{4/5}]$ for m. The test statistic should be insignificant under H_0 and significantly positive if the underlying process is short memory with level shift.

The other two tests attempt to measure whether there is a sharp decline in $\hat{d}$ as m increases from $T^{1/3}$ to $T^{1/2}$. As it is uncertain for which m the estimate $\hat{d}$ is at a maximum, the tests take the form of sup-$t_d = \sup_{c_1 \epsilon [1,2]} t_d\left(\frac{1}{3}, c_1; \frac{1}{2}, 1\right)$ and *mean*-$t_d = mean_{c_1 \epsilon [1,2]} t_d\left(\frac{1}{3}, c_1; \frac{1}{2}, 1\right)$. The limit distributions of the sup-t_d and *mean*-t_d tests are unknown but assuming an ARIMA$(1, d, 1)$ process, five per cent bootstrap critical values are around 2.5 and 1.6, respectively. If the underlying process is short memory with a level shift, then both test statistics should be significant and positive.

3.3 Shimotsu's 'simple but effective' tests

Of the suite of tests introduced by Shimotsu (2006) to help discriminate between the options of fractional integration and nonlinearity of a time series, the first is a 'Wald' test derived by comparing the estimate of d for the entire time period with the estimates of d for subperiods. The other two tests make use of the behaviour of the standard Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) and Phillips-Perron tests of the d-differenced series.

More formally, the first test is a test of the null hypothesis H_0: $d_0 = d_{0,1} = \ldots = d_{0,n}$, where d_0 is the true value of d for the entire sample, and $d_{0,i}$ is the true value of d from the i^{th} subsample. Using either the exact local Whittle (*ELW*) or the two-step feasible exact local Whittle (*FELW*) estimator $\hat{d}$ of d_0, the Wald statistic for testing H_0 is

$$W_c = 4m\left(\frac{c_{m/n}}{m/n}\right) A\hat{d}_n \left(A\Omega A'\right)^{+} \left(A\hat{d}_n\right)', \tag{7}$$

where

$$\hat{d}_n = \begin{pmatrix} \hat{d} - d_0 \\ \hat{d}^{(1)} - d_0 \\ \vdots \\ \hat{d}^{(n)} - d_0 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & -1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \ldots & -1 \end{pmatrix},$$

$\left(A\Omega A'\right)^{+}$ denotes a generalised inverse of $A\Omega A' = nI_n - i_n i_n'$ (where I_n is the identity matrix of order n and i_n is the unit n-vector), and c_m is a small-sample correction factor to allow for the larger size of the finite sample variance:

$$c_m = \sum_{i=1}^{m} v_i^2, \qquad v_i = \log i - \frac{1}{m}\sum_{i=1}^{m} \log i.$$

Under H_0, W_c has an asymptotic χ^2 distribution with $n-1$ degrees of freedom and the usual decision criterion applies.

The other tests are based on the behaviour of the d-differenced series. If an $I(d)$ series is differenced $\hat{d}$ times, where $\hat{d}$ is a consistent estimate of d, the resultant series should be $I(0)$. Shimotsu considers two tests: Phillips and Perron's (1988) Z_t for the partial sum process of the differenced series, and the KPSS test for the differenced series. The value of d used in the differencing is that given by the *FELW* estimator. To allow for bias caused by the short-run dynamics of the series, the differencing is carried out on a mean-adjusted series. The adjustment used by Shimotsu, assuming a series $X = \{X_1, X_2, \ldots, X_T\}$, is

$$\mu(d) = w(d)\overline{X} + (1 - w(d))X_1, \tag{8}$$

where $\overline{X}$ is the sample arithmetic mean, and $w(d)$ is a smooth (twice continuously differentiable) weight function such that $w(d) = 1$ for $d \leq \frac{1}{2}$ and $w(d) = 0$ for $d \geq \frac{3}{4}$. The $w(d)$ used by Shimotsu when $d \in \left(\frac{1}{2}, \frac{3}{4}\right)$ is $\frac{1}{2}[1+cos(4\pi d)]$. The asymptotic distribution of these two test statistics depends on d, and simulated critical values are provided in Shimotsu (2006).

4 Results and Discussion

Table 1 reproduces the relevant parts of Bond, et al.'s (2007) fractional integration analysis using the FADF test.[2] For each of the two real exchange rate series, four different estimates of d are given, together with their estimated standard errors and, where computed, their associated FADF test statistic values. The FADF test is only meaningful, and hence reported, if $d < 1$, in which case the critical values for judging the test statistics are the standard normal ones. The two parametric estimates of d in the table are the exact maximum likelihood (*EML*) estimate and a nonlinear least squares (*NLS*) estimate. The *EML* estimate is computed using the algorithm suggested by Sowell (1992). The *NLS* estimator is an approximate maximum likelihood estimator developed by Beran (1995) and based on the conditional sum of squared naïve residuals. The other two estimates of d presented derive from the nonparametric log periodogram *GPH* method and the Gaussian semiparametric (*GSP*) method, both being calculated using the square root of the sample size for the number of frequencies. For both series, the various estimates of d lead to conflicting conclusions, although there is a strong suggestion of a unit root in the Ireland-UK real exchange rate. The FADF test provides strong evidence of fractional integration in the case of the Ireland-Germany real exchange rate only when the *GPH* and *GSP* estimates of d are used.

Table 2 presents the Bond, et al. (2007) nonlinearity test results for the real exchange rates,

[2] All tables, and figures, are in the Appendix.

produced by RESET, STAR and random field-based procedures.[3] For all the tests, the null hypothesis is that each series is linear. For the RESET test, both the F and likelihood ratio (LR) variants are given. For the STAR tests, the standard F-tests are used; see Lütkepohl and Krätzig (2004). The Akaike information criterion suggests a lag length of three for the STAR tests in the case of the Ireland-Germany exchange rate and a lag length of two for the Ireland-UK case. The Schwarz information criterion suggests a lag length of one in both cases. All three sets of tests suggest that the assumption of linearity is adequate for the Ireland-UK real exchange rate. However, whereas the random field tests overwhelmingly support linearity of the Ireland-Germany real exchange rate, the STAR test based on the use of three lags gives some indications of nonlinearity and the RESET test rejects linearity very decisively. Bond, et al. (2007) noted these confusing results and went on to the causal modelling of the nominal exchange rate, for which there were much clearer indications of nonlinearity. Here the alternative route of subjecting the real exchange rates to further scrutiny using the new procedures described in the previous section is taken.

Table 3 gives the *GPH* estimates, $\hat{d}$, and the *modified-GPH* estimates, $\hat{d}^k$, for the two real exchange rate series.[4] For each series the parameter is estimated using both the root mean square error minimising 'Plugin' value and the $T^{1/2}$ value for m, and a range of values for k. For both series there is considerable evidence in favour of nonlinearity, $\hat{d}^k$ being less than $\hat{d}$ in the large majority of cases. For both series, when m is set to the 'Plugin' value, $\hat{d}^k < \hat{d}$ for all cases except $k = 4$ (and $k = 5$ for Germany). Using the fixed $m = T^{1/2}$, the indications are that the real exchange rate between Ireland and Germany is nonlinear and that between Ireland and the UK is not. Therefore the *modified-GPH* test supports the findings of Bond, et al. (2007) with respect to nonlinearity of the real exchange rate between Ireland and Germany but also suggests the possibility of nonlinearity in the real exchange rate between Ireland and the UK, which was much less clear in their results. Moreover, the earlier indications of fractionality in Table 1 may well be spurious.

Figure 2 plots the values of the *GPH* estimates of d as the frequencies vary from $T^{0.3}$ to $T^{0.8}$ in steps of 0.01. Like the time series of the two real exchange rates, these two plots exhibit quite different behaviour. For the Ireland-UK real exchange rate, the shape of the graph is similar to that discussed in Perron and Qu (2008), though the values of $\hat{d}$ are much higher. In the case of the Ireland-Germany rate, the value of $\hat{d}$ seems to slowly rise as the number of frequencies increases.

Figure 3 displays the values from the Perron-Qu $t_d(a, 1; 0.8, 1)$ test for $0.3 \leqslant a < 0.8$. Also

[3]Details of all the nonlinearity tests and the notation used in Table 2 are given in Bond, et al. (2007).

[4]The calculations were done using Smith's GAUSS code, available from `http://www.agecon.ucdavis.edu/people/faculty/info.php?id=32.`

plotted are the approximate five per cent critical values. Interestingly, it is the test for the real exchange rate between Ireland and the UK that suggests the possibility of nonlinearity, rather than that for the real exchange rate between Ireland and Germany. Table 4 gives the results of the Perron-Qu tests. In the case of both exchange rates, the $t_d\left(\frac{1}{2}, 1, \frac{4}{5}, 1\right)$ test fails to reject the null hypothesis of long memory. The sup-t_d and *mean*-t_d tests support the possibility of nonlinearity in the case of Ireland-UK (the former at the five per cent significance level and the latter at the ten per cent level) but fail to reject the hypothesis of long memory for Ireland-Germany. However, in the first case the values of $\hat{d}$ are greater than one for the range of tests.[5]

The results of the Shimotsu analysis of the two real exchange rates are shown in figures 4 and 5; the five per cent critical value is also provided in each plot.[6] The choice of five breaks is based on the analysis of Bond, et al. (2007). For the real exchange rate between Ireland and the UK, the Wald statistic, W_c, gives some support for the possibility of five breaks at the five per cent level, as it is close to the critical value 11.07 for $m > T^{0.6}$. If the ten per cent significance level (critical value 9.23) is used, the support is of course deemed stronger. The W_c test also gives some support for the possibility of one or five breaks for the real exchange rate between Ireland and Germany at the five per cent significance level. By contrast, the KPSS test for both series supports the null of long memory. The results of the Phillips-Perron Z_t test are the reverse of the results of the W_c test. They strongly support the possibility of nonlinearity in the real exchange rate between Ireland and Germany but give little support for that possibility in the case of the Ireland-UK rate. The choice of the number of breaks does seem to influence the outcome of the W_c test. Figure 6 shows the results of the W_c test for three breaks, but the test statistic is not significant at any frequency for either of the real exchange rate series in this case.

5 Conclusions

In this paper, three new approaches to investigating the complex relationship between nonstationarity and nonlinearity have been described and applied to the log real exchange rates between Ireland and the UK and Ireland and Germany. Unlike Bond, et al. (2007), who found little indication of nonlinear behaviour in these real exchange rates – despite strong evidence in favour of nonlinearities in the corresponding nominal time series – the new tests suggest a strong possibility

[5]The calculations for all Perron-Qu procedures were done using GAUSS computer code written by Derek Bond.

[6]The calculations were done using Shimotsu's computer code from `http://www.econ.queensu.ca/faculty/shimotsu/`. The differences between the *GPH* estimates for fixed m reported in Table 3 and those in Table 1 are due to the fact that Bond, et al. (2007) used a different algorithm in the ARFIMA package of Doornik and Ooms (1999).

that both manifest some nonlinear behaviour. To this extent, the original Bond, et al. (2007) findings of possible long memory in the series may well be spurious. On the other hand, the new Shimotsu KPSS procedure appears to signal long memory quite clearly.

As Figure 1 shows, the two real exchange rate series follow quite different time paths and this difference in behaviour could be the reason why the various tests do not all concur. The Ireland-Germany series is much smoother than the Ireland-UK series. The new tests produce copious amounts of output when the number of frequencies, m, used in the estimation of the fractional differencing parameter, d, is varied. Rather than rely on a few particular values of m, estimates of d, and test statistics, have been calculated for a large range of values of m and the results graphed. Table 5 attempts to summarise these results and the main points discussed below. With regard to the terms used in Table 5, 'Probable' connotes a somewhat greater likelihood than 'Possible'.

Smith's (2005) *modified-GPH* procedure, which is probably the most attractive of the three approaches in practice because of its rather more general underpinning model, strongly supports the case for nonlinearity of the real exchange rate between Ireland and Germany; it also indicates the same for Ireland and the UK, if the root mean square error minimising value of m is used. As Figure 2 shows, the estimated value of d varies more for the Ireland-UK real exchange rate than it does for the Ireland-Germany rate over the period studied, and it would seem that the outcome of the test can depend on the choice of m.

The performance of the $t_d(a, c_1; b, c_2)$ tests proposed by Perron and Qu (2008) are interesting. When comparing low frequency to high frequency *GPH* estimates of d, the tests suggest that the real exchange rate between Ireland and the UK might be nonlinear. However, they fail to produce any indication of nonlinearity for the real exchange rate between Ireland and Germany. It should be stressed, though, that these tests are based on a null hypothesis of stationary long memory ($d < 0.5$). The accumulating evidence suggests that if Irish real exchange rates are long memory, they are nonstationary processes with $d \geqslant 0.5$.

The results of the various Shimotsu (2006) tests are also interesting. The W_c (Wald) test gives some support for the nonlinearity of the Ireland-UK real exchange rate when the possibility of five breaks is considered, and some support for either one or five breaks for the Ireland-Germany series. The KPSS test, as already mentioned, does not support the possibility of nonlinearity for either series, suggesting instead that fractionality may be an appropriate reason for their behaviour. The inferences from the Phillips-Perron Z_t test are the reverse of those suggested by the W_c test.

There does, therefore, seem to be enough evidence from these new procedures to suggest that despite the Bond, et al. (2007) findings, the two real exchange rates might be nonlinear over time and that certain of their findings regarding long memory may be spurious. However, the results of the Shimotsu KPSS test supporting the possibility of long memory may be important. Without knowing more about the relative power of the tests, their robustness to departures from the assumptions specified under the respective null hypotheses, and their performance against alternative forms of nonlinearity to those specified in the tests, it is difficult to draw any strong conclusions. There is little doubt that the performances of the new tests are related to the precise behaviour of the underlying series they are applied to.

Thus, although some additional insights have been provided by the new tests, the nature of the real exchange rate time series examined is still unclear. The conclusion is emerging that their data generating processes might be more complex than either pure long-memory processes or stationary processes subject to structural change. As Perron and Qu (2008, p. 18) point out, when estimates of d change with different values of m but remain above 0.5, as in the present study, the indications are of long-memory processes with level shifts. This might well be the reason for the results reported in this paper. Whether or not this is so, the challenge of modelling Irish real exchange rates adequately remains.

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A Appendix

A.1 Tables

Table 1: Fractional Integration Analysis

Countries	*EML*	*NLS*	*GPH*	*GSP*
			FADF	
Ireland-Germany	1.41 (0.08)	1.48 (0.08)	0.98 (0..11)	0.85 (0.07)
	-	-	-5.05	-5.12
Ireland-UK	1.07 (0.09)	1.08 (0.09)	1.15 (0.11)	0.97 (0.07)
	-	-	-	-1.09

Note: standard errors in parentheses.

Table 2: Nonlinearity Tests

Test		Test Statistic	P-value	Bootstrap p-value	Test Statistic	P-value	Bootstrap p-value
			Ireland-Germany			Ireland-UK	
Reset							
	F	8.136	0.000		1.043	0.376	
	LR	23.606	0.000		3.969	0.349	
STAR				Lag length 1			
	F		0.236			0.576	
	F4		0.379			0.952	
	F3		0.121			0.169	
	F2		0.303			0.764	
			Lag length 3			Lag length 2	
	F		0.010			0.207	
	F4		0.054			0.108	
	F3		0.010			0.236	
	F2		0.039			0.591	
Random Field							
	$\lambda^E_{H(g)}$	2.410	0.121	0.058	0.187	0.665	0.653
	λ^A_{OP}	4.481	0.923	0.369	6.721	0.751	0.394
	$\lambda^E_{OP(g)}$	0.035	0.852	0.922	1.056	0.304	0.562
	g_{OP}	4.551	0.871	0.367	2.847	0.970	0.458

Table 3: *Modified-GPH*: Log Real Exchange Rates

	m	*GPH*	*Modified-GPH* $k=1$	$k=2$	$k=3$	$k=4$	$k=5$
UK	Plugin	0.91 (0.18)	0.89 (0.64)	0.84 (0.4)	0.86 (0.31)	1.1 (0.27)	0.87 (0.23)
	Fixed	1.2 (0.29)	2.4 (2)	2 (1.1)	1.9 (0.87)	1.8 (0.76)	1.8 (0.71)
Germany	Plugin	1 (0.18)	0.67 (0.64)	0.85 (0.4)	0.92 (0.31)	1 (0.27)	1 (0.23)
	Fixed	0.89 (0.29)	0.67 (2)	0.74 (1.1)	0.75 (0.87)	0.76 (0.76)	0.76 (0.71)

Note: standard errors in parentheses.

Table 4: Perron-Qu $t_d(a, c_1, b, c_2)$ test

$t_d(0.5, 1; 0.8; 1)$	Sup-t_d	$Mean$-t_d
Ireland-Germany		
-0.934	0.000	-0.313
Ireland-UK		
0.505	3.951	1.215

Table 5: Summary of Results
(Likelihood of the series being nonlinear)

Test		Ireland-UK	Ireland-Germany
Smith's *Modified-GPH*			
	Plugin	Probable	Probable
	Fixed	No Support	Probable
Perron-Qu's $t_d(a,1;b,1)$			
		Possible	No Support
Shimotsu's			
	Wald W_c		
	One break	No Support	Possible
	Five breaks	Possible	Possible
	KPSS	No Support	No Support
	Phillips-Perron Z_t	No Support	Probable

A.2 Figures

Figure 1: Time Series of Log Real Exchange Rates

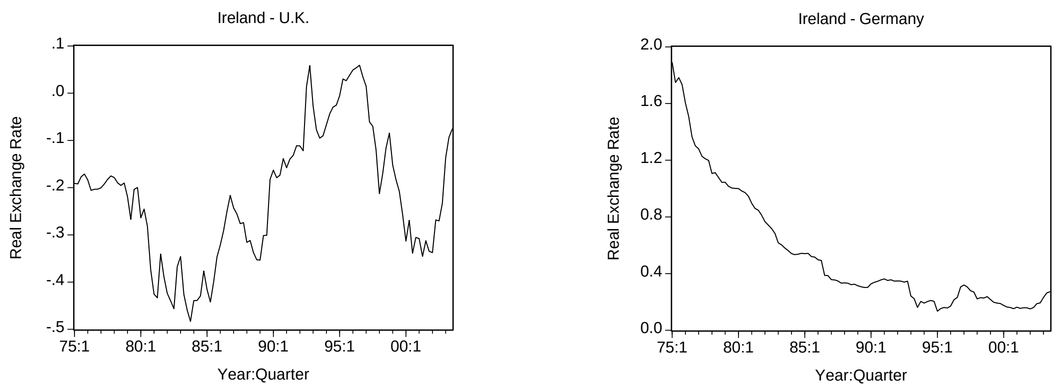


Figure 2: Estimate of d using GPH method with T^a frequencies

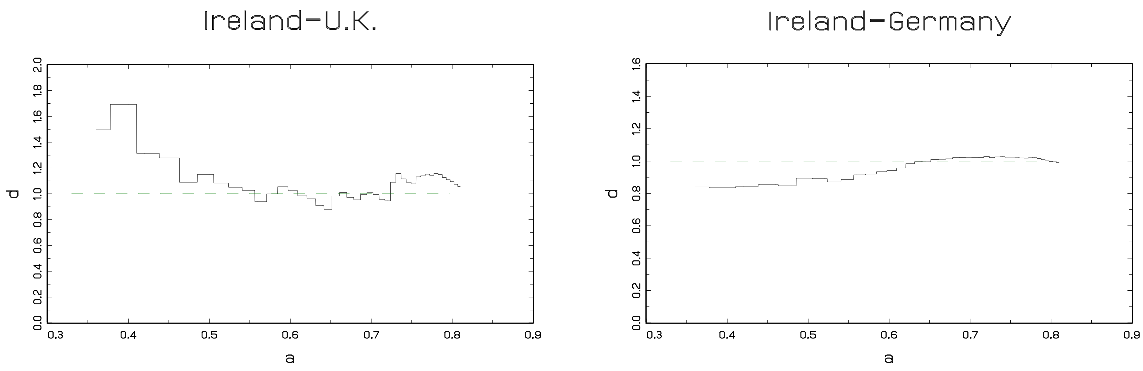


Figure 3: Perron-Qu $t_d(a, 1; 0.8, 1)$ test

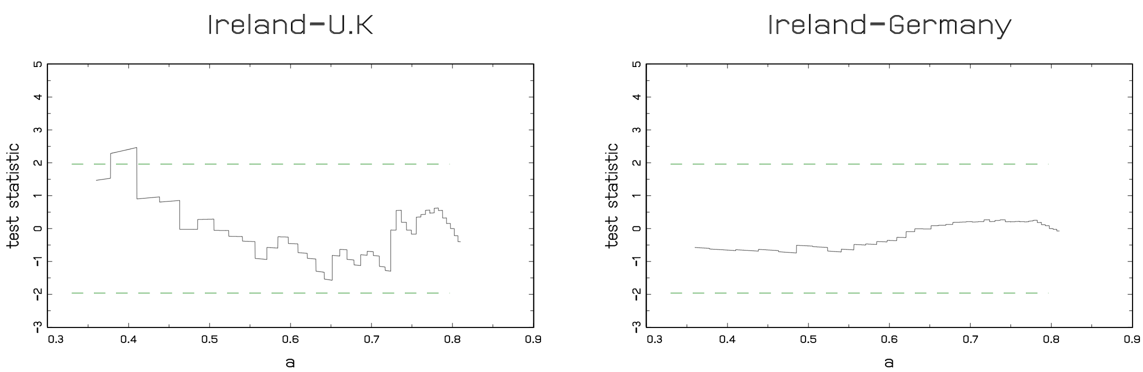
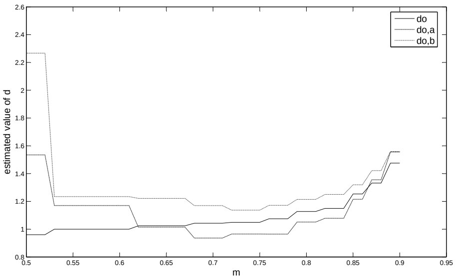
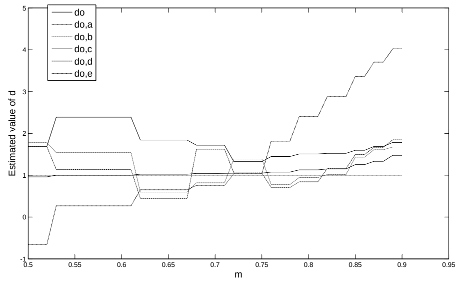


Figure 4: Shimotsu Analysis: Ireland-UK

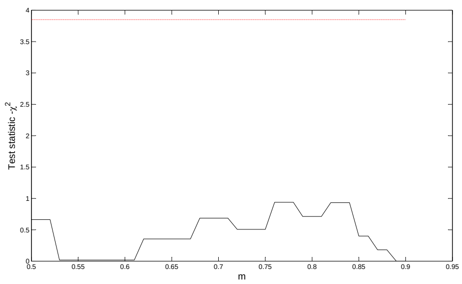
One break | Five breaks

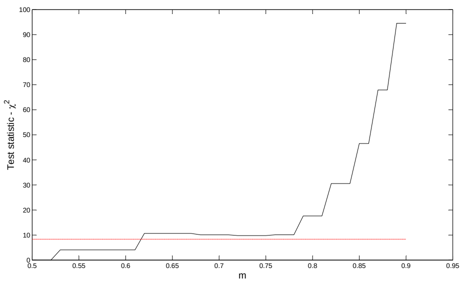
Estimates of d



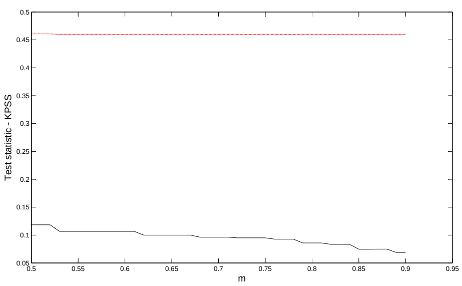


W_c Wald test





KPSS test | Phillip-Perron test



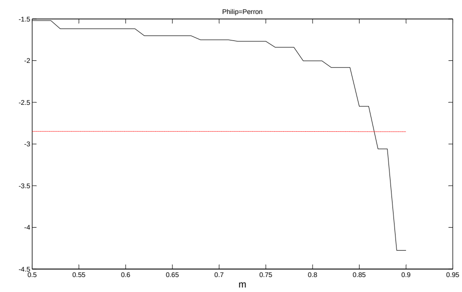
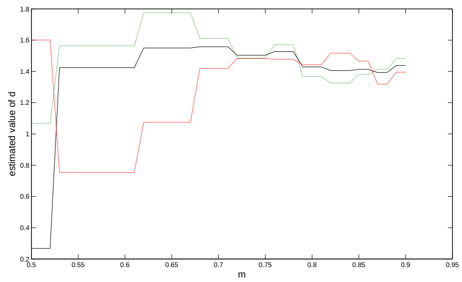
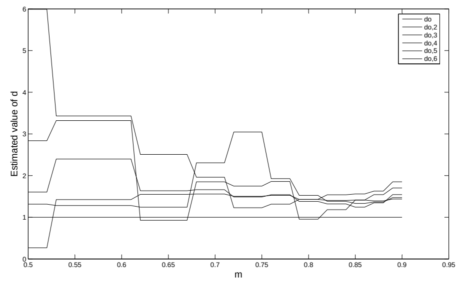


Figure 5: Shimotsu Analysis: Ireland-Germany

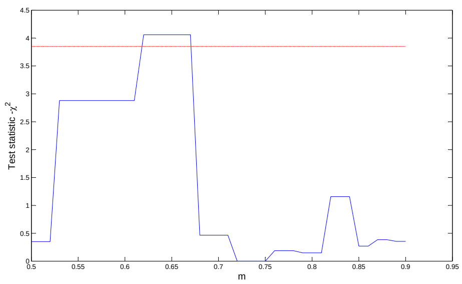
One break | Five breaks

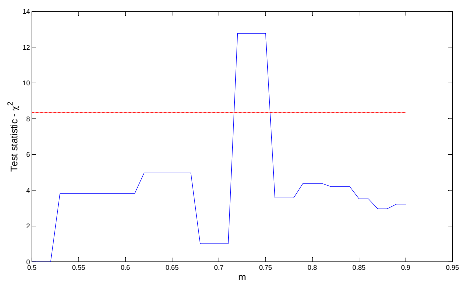
Estimates of d



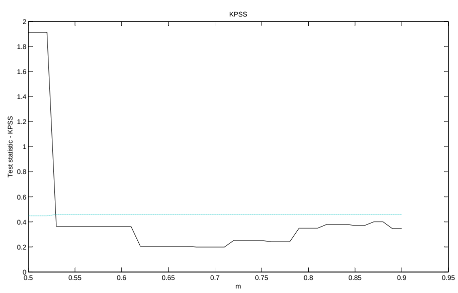


W_c Wald test





KPSS test | Phillip-Perron test



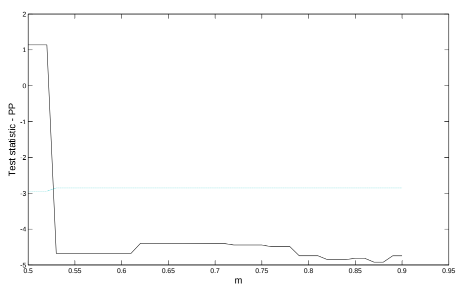
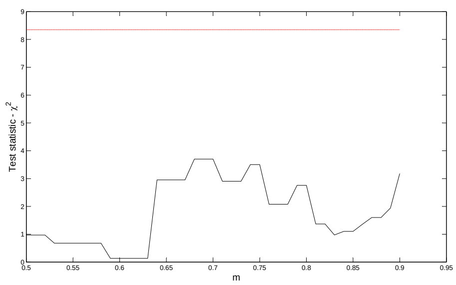


Figure 6: Shimotsu's W_c Wald Test for 3 Breaks

Three breaks
Ireland-UK



Three breaks
Ireland-Germany